Swedish Institute for Social Research (SOFI)

Stockholm University

WORKING PAPER 1/2013

PRICE FORMATION IN CONSUMER MARKETS

by

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Price Formation in Consumer Markets*

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January 30, 2013

Abstract: This paper argues that a completely non-cooperative approach to pricing is neither necessary nor plausible in consumer markets. It proposes instead a model where firms decide non-cooperatively on production or marketing, while the market price is set by a competitive price leader, i.e. a firm preferring the lowest market price. Predictions include a revenue-maximizing market price and excess supply in markets where production precedes sales, and non-monopolistic pricing if firms are 'sufficiently dissimilar' in markets where sales precede production. The model also provides a simple and plausible explanation of why markets do not always clear.

Keywords: Pricing, oligopoly, price leadership, market shares, marketing
JEL classification: D43, L13

* Discussions with Jörgen W. Weibull and Henrik Horn during the first phase of the research reported here were very valuable. I would also like to thank Mats Bergman and participants of seminars at Stockholm University and Åbo Akademi University, and in particular Jim Albrecht, Mahmood Arai, Torsten Persson, Rune Stenbacka, Lars E.O. Svensson, Susan Vroman, Eskil Wadensjö and Johan Willner for useful comments on earlier versions.

1. Introduction

The market-clearing paradigm is still *the* bench-mark in introductory textbooks in economics. And this model certainly makes sense in financial markets with daily market clearing and volatile prices.[1] But it has not been used in oligopoly theory, which so far has been built on Cournot models (for markets where production precedes sales) and Bertrand models (for markets where sales precede production).[2] This paper argues, however, that there is a simple modification of the traditional model with price-taking agents which makes sense in markets where buyers take prices as given and prices are set by firms, even more sense than Cournot models and Bertrand models. For, even if not all firms can be price takers, all firms but one can take the price as given. And by replacing an imaginary auctioneer with a real price leader maximizing its individual profits, we obtain a consistent and tractable model with plausible assumptions and realistic predictions, including markets which do not always clear and competition in other variables than prices.

In this model the market price goes down if and only if a price cut appears profitable for a firm *even if its competitors follow suit*, while the market price goes up if and only if a higher market price is profitable for every firm. Thus, the market price is determined by the lowest market price preferred by a firm, an idea which goes back at least to Boulding (1941 p. 610).

When all firms prefer the same market price, the choice of price leader is immaterial. It might be thought that the market price always is monopolistic in this case, but we shall see that competition in other variables than prices can imply a lower market price. And when price preferences differ, the price leader is the firm preferring the lowest market price, which sometimes may approach even competitive levels. Note that a price leader in this paper is not a collusive price leader (maximizing its industry's profits) but a competitive price leader (maximizing its individual profits).

To model price formation we have to distinguish between different types of markets, as emphasized, for instance, by Okun (1981 ch. 4), Shapiro (1989), and Vives (1999 ch. 10). And my message is that while the Cournot model is applicable to (some) auction markets and the Bertrand model is applicable to bidding markets, the Boulding model is applicable to price-taking markets, which I define as markets where buyers take prices as given and prices are set by sellers, as in most consumer markets.

[1] In practice market clearing is often realized by 'match makers' matching offers to sell to offers to buy, as in markets for stocks, or by 'market makers' buying and selling securities at offered prices and adjusting prices so that supply equals demand during a day, as in markets for foreign exchange.
[2] See, for instance, Vives (1999) for a modern exposition and discussion of oligopoly theory, including its problems of indeterminacy, especially in repeated games.

Price-taking markets are characterized more precisely in Section 2, and my arguments for price leadership in such markets are developed in Section 3. Some examples of the consequences of the Boulding model are then given in Section 4 for markets where production precedes sales and in Section 5 for markets where sales precede production. Results are summarized in Section 6 and related to the management literature in Appendix 1.

In this paper price leadership is developed within the framework of “no side payments and partial preplay communication”, which Luce and Raiffa (1957 p. 169) once characterized as the most surprising omission in the literature on games. In fact I exclude all binding agreements and I also exclude all communication apart from observation of prices.

When consumers take prices as given there is a well-defined demand function which determines what consumers buy at a given market price. We assume that this demand function is decreasing in the market price (so that its inverse exists) and that its price elasticity is non-decreasing and greater than 1 for some price (so that a monopoly price is finite and unique). And with these assumptions there will be no problems of existence or uniqueness giving rise to indeterminacy, as in, for example, Bertrand-Edgeworth models.

2. Price-taking markets

We focus on markets where buyers take prices as given in this paper and especially on consumer markets.We also focus on the short run, when firms, costs, capacities and other market conditions can be taken as given during some time, which I call a market period (for example a year or a quarter). Prices will of course depend on costs and capacities, and in general there will also be an important trade-off between variable costs and fixed costs, but I will not study investment decisions in this paper.

Pricing is indeterminate until we have specified and motivated a market form, that is, a set of rules for the pricing game. In this section I characterize price-taking markets by introducing some plausible assumptions and then propose, in Section 3, a market form which is consistent with these assumptions.

ASSUMPTION 1. Buyers are price takers.

By assuming that buyers take prices as given we assume that buyers can observe prices at any time but we also exclude indeterminacy due to haggling or bargaining. The exclusion of haggling reduces transaction costs and facilitates price comparisons. By excluding bargaining we exclude the possibility for buyers to bargain with producers over prices, as in many business-to-business markets or markets with ‘buying groups’.

ASSUMPTION 2. While prices are set by sellers, buyers are free to choose among sellers.

Thus we exclude the possibility for producers to fix market shares and implicitly postulate the existence of a competition authority which can enforce this rule of the market game, excluding, for instance, the possibility for firms to allocate market shares through a common sales organisation.

Assumption 2 implies that market sharing in general is determined endogenously by competition between firms for market shares, as exemplified in Sections 4 and 5.

ASSUMPTION 3. Prices are set in the beginning of the market period after a short period of price adjustment when firms can observe and revise their prices at any time.

In a market where buyers take prices as given, trade cannot start until prices have been announced by the sellers. Preliminary list prices may be announced independently by firms in the beginning of a market period – possibly according to cost-plus pricing or value-based pricing as elaborated in Appendix 1 – but firms are not committed to these prices as in sealed bidding (and Bertrand games). Instead firms can observe their competitors' prices and adjust their own if they want to. I interpret the end of the adjustment process as acceptance: price adjustment does not end until all firms accept competitors' prices by not reacting to them.

This assumption means that all consumers can take prices as given (constant) during the market period, which greatly facilitates their planning. It also means that all firms take prices as given during the market period, as in the traditional model of perfect competition. However, I do *not* assume that price-taking firms can sell all they want to produce at the given prices. Thus, I do not exclude by assumption the possibility that production is restricted by sales at prices chosen by firms.

ASSUMPTION 4. Price differentials are negligible (the law of one price).

If products are perfect substitutes, consumers will buy the cheapest product as long as its producer is not restricted by its capacity. By assuming in addition that a capacity-constrained producer with excess demand raises its price, price adjustment will imply convergence towards a uniform market price in the beginning of the market period.

The concept of a market price is fundamental to the concept of a market and is applicable also to markets with differentiated goods, even if it then has to be interpreted as a measure of the price level. Thus, if price differentials persist, due to heterogeneity or switching costs, we define the market price as the average of all prices (perhaps weighted with firms' market shares) or as the price of a dominant firm if there is one. Deviation from the market price can be interpreted, for instance, as compensation for quality above (or below) the average. But note that this paper, when applied to product differentiation, does not attempt to explain price differentials, only the price level.

I consequently assume that pricing in practice can be separated into two problems: price level and price differentials. A market price (price level) changes regularly, when market conditions change, while price differentials do not necessarily change over time. In some markets price differentials may be small and constant over time, and then a theory of the market price is pretty exhaustive. In other markets price differentials may be so large and variable that they require additional explanations. These can be more or less intuitive in terms of ‘history’ or ‘quality’. Or price differentials can be explained as a self-enforcing agreement, but such an explanation presupposes that individual demand functions are not only well-behaved but also well-known to every firm in the market.

ASSUMPTION 5. There are no side payments between firms.

Thus, firms preferring a high market price cannot persuade a firm preferring a low market price from setting this price. More precisely, we postulate the existence of a competition authority which can prevent firms from making agreements involving side payments.

Assumptions 2 and 5 are necessary because in a completely unregulated market, where buyers take prices as given and prices are set by sellers, any number of collusive outcomes is possible and pricing consequently indeterminate.

ASSUMPTION 6. Firms want to maximize profits in the short run.

We consequently assume that prices are set by firms anticipating the consequences for sales, production and profits during the immediate market period – but only during this period.

Of course, anticipated consequences in future market periods may sometimes matter for the choice of prices set in the current market period. If, for example, existing firms believe that high prices now may attract new firms in the future and reduce profits in the long run, they may ‘limit’ their prices, as in, for example, Judd and Petersen (1986). Another example is that firms predicting that existing market structure will persist more or less indefinitely may be tempted to set high prices, because they believe that repeated interaction will foster ‘collusive’ behaviour, as in, for example, Rotemberg and Saloner (1990). But these examples also show that price formation which involves expectations of the distant future should be dealt with separately, since such expectations may be so different.

In any case, maximizing profits in the long run is a difficult dynamic problem, even for a monopolist but in particular for a firm which has to anticipate the strategies of current and potential rivals. And note that such a problem cannot be modelled correctly as a dynamic game without a correct representation of the market form for each stage of the market process.

Note also that the consequences for current price formation of profit-maximization in the long run can fruitfully be expressed as deviations from a 'standard model' based on Assumption 6. Moreover, this assumption is not always restrictive, since firms often focus on short-term profits for various reasons, like financial restrictions, stiff competition, demanding share-holders, or a declining industry. A prediction based on Assumption 6 can also be interpreted as a first approximation whenever long-term objectives or strategies are unknown.

3. Price adjustment and price leadership in price-taking markets

It remains to specify exactly how initial price announcements are made, how prices are adjusted, and how the market price is established in the beginning of the market period, before trade takes place.

Bhaskar (1989) gives a precise formulation of Assumption 3 by assuming that price decisions p_i are taken at time *t* by firm *i*, and set equal to the price announcement at time *t*, $p_i = p_i(t)$, if and only if, for every *j*, $p_j\left(t\right) = p_j\left(t-1\right)$. Price announcements are consequently not perceived as final price decisions (trading prices) until, after having been observed, they are repeated by every firm.

We interpret repetition as acceptance. Every firm can veto or 'vote against' the current price vector merely by changing its own price. On the other hand, a firm accepts or 'votes for' the current price vector by not changing its own price. Price announcements become price decisions when accepted by every firm in this sense. Moreover, since no firm is committed to its initial price announcement, it is not restrictive to assume that $p_i\left(0\right) = p_i^o$, where p_i^o denotes the market price preferred by firm *i*.

Consider for simplicity a duopoly and define price-taking behaviour for firm *i* by the pricing strategy $p_i\left(t+1\right) = p_j\left(t\right)$, $t = 0,1,...$ .Then it is easy to see, when the firms prefer the same market price, $p_1^o = p_2^o = p^o$, that price-taking strategies (with preferred prices as initial price announcements) constitute a Nash equilibrium with $p_i = p^o$ as final price decisions.

On the other hand, if firms can agree on playing a non-cooperative dynamic game with rules as specified above, they should also be able to agree on price leadership when they prefer the same market price.[3] And price leadership is not only a simpler market form but also easy to motivate directly and apply to situations where price preferences differ.

3.1 Definition of price leadership

In this paper price leadership means that all firms but one take the price as given or, more precisely, that one of the firms sets a price which the other firms match. This form of price leadership is also frequently documented and discussed in classical literature, including Boulding (1941), Stigler (1947) and Markham (1951).[4] And while setting the same price as another firm suggests collusion in markets with sealed bidding, it is both possible and legal in markets where firms are free to observe and revise their prices at any time.[5]

In classical writings a price leader is often a dominant firm. Markham (1951 p. 895-896) notes that "nearly every major industry in the American economy has, in its initial stages of development, been dominated by a single firm", and that "the monopoly power of the initial dominant firm in most industries […] was gradually reduced by industrial growth and the entrance of new firms". Hence it is easy to understand why 'dominant firm' price leadership dominates the classical literature and why it often is called 'partial monopoly'.

Assuming that new firms take the dominant firm's price as given and produce competitively, it is also easy to see why the dominant firm, when maximizing its profits conditional on its residual demand curve, will set a price which is decreasing in the total capacity of the "competitive fringe" (Scherer 1980 p. 233). However, the assumption that the dominant firm *always* sets a price at which the other firms produce competitively is too restrictive, as we shall see in Section 5.

To derive the identity of a price leader and the outcome of price leadership I start from Boulding (1941 p. 607-613). His basic idea is that if firms prefer the same market price, then the choice of price leader is immaterial and the outcome is monopolistic, while if firms prefer different market prices – due to differences in costs or capacities or market shares – then a firm preferring the lowest market price will be a *competitive* price leader, to use the term introduced by Lanzillotti (1957).

Price leadership has often been divided into three categories, namely dominant firm, collusive and barometric (Scherer 1980 p. 176), but without precise definitions. I find it

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[3] This argument also applies to a non-cooperative game designed to endogenize price leadership.
[4] Recent articles on price leadership, e.g. Deneckere and Kovenock (1992) and Pastine and Pastine (2004), focus instead on Stackelberg price leadership, meaning that the followers optimize against the price set by a price leader committed to its price.
[5] Moreover, while a firm copying a competitor's patent or design is often sued by the competitor, so far a firm setting the same price as another firm has not been sued.

clarifying, to begin with, to distinguish between *collusive price leadership*, where the price leader is assumed to maximize its industry's profits, and *competitive price leadership*, where the price leader is assumed to maximize its individual profits. And I will here focus on competitive price leadership.

More precisely, I define a *competitive price leader* as a price leader preferring the lowest market price. If there are many such firms, the choice of price leader among these is immaterial and may be expected to vary randomly or depend on which firm is assumed to have the best information on market conditions. A competitive price leader may in this case also be called a *barometric price leader*, following Stigler (1947). And if there is only one firm preferring the lowest market price, it may be called a *dominant price leader*.

Moreover, excluding the possibility of side payments, firms preferring a high market price cannot persuade a firm preferring a low market price to abandon that market price which maximizes its individual profits. And the firm preferring the lowest market price can implement it simply by announcing it, while firms preferring a higher market price are forced to follow suit, at least if the price leader has excess capacity.[6]

3.2 Observation of price leadership

How can price leadership be observed? A necessary condition, which is particularly easy to observe, is that there is *not* an auctioneer in the market or a big buyer enforcing sealed bidding. A sufficient condition is that, when market conditions change, price adjustment is initiated by one of the firms and followed by the other firms in the market. And if firms simultaneously announce new list prices, price leadership is also obvious if some firms adjust their prices to another firm's price after the initial announcement.

But what if firms simultaneously announce new list prices and there is no adjustment at all? Since firms are free to adjust their prices if they want to, price differentials must be not only small (because of the law of one price) but also acceptable to every firm. This means that we can interpret the outcome 'as if' the market price (price level) is determined by a barometric price leader when all firms prefer the same market price. It cannot be interpreted as the outcome of a price cartel (maximizing joint profits) unless a firm objects to the price agreement and would have preferred a lower market price.

[6] If not, i.e. if the price leader prefers a market price which clears the market, then it may appear profitable (at least in the short run) for another firm to raise its price. However, this will not reduce the profits of the price leader, who consequently may choose not to react to the price rise. This suggests the possibility of some price dispersion, but it also shows that no (pure) non-cooperative equilibrium exists in market-clearing prices.

3.3 Consequences of price leadership

A fundamental stylized fact in consumer markets is that firms set prices at which they share the market – in contrast to sealed bidding where only the cheapest bidder can sell. And if firms share the market, prices must be (approximately) the same, according to the law of one price. It follows that rational firms will have preferences on the market price, including market prices above the market-clearing level. But then firms can no longer stick to the presumption that they can sell everything they produce. Instead they will realize that there is rationing and they will adapt to this.

But how are producers rationed, or, more precisely, how will the market be shared when production is restricted by sales and not capacity? This fundamental question is addressed in Section 4 for markets where production precedes sales (where excess supply may arise in equilibrium after output adjustment), and in Section 5 for markets where sales precede production (where excess supply is excluded by definition). At a given market price a firm will in general attempt to increase its market share by *marketing*, defined as measures which by creating additional value to customers increase the firm's sales. Hence market shares are in general determined endogenously by a non-cooperative game in marketing. A firm may also attempt to increase its market share by cutting its price, but a rational firm will do this only if it increases the firm's profits even if its competitors follow suit.[7]

4. Pricing and production in markets where production precedes sales

In markets where production precedes sales producers have to 'bring their goods to a market place' before sales are completed. The traditional approach to this problem in oligopoly theory is based on the Cournot model, where firms choose quantities and a market-clearing auctioneer or process sets the market price. This is also an adequate model in markets for agricultural products and oil and other raw materials, where an auctioneer sets that price which equals demand to that supply which has been brought to the market place. But it is not applicable to markets where buyers take prices as given and prices are set by sellers, as in most consumer markets. In such markets firms choose quantities *and* prices.

A strictly non-cooperative approach to this decision problem corresponds to a market form where firms simultaneously and independently commit to both prices and quantities, implying sealed bidding on prices. Some results on equilibria for this market form are also

[7] In other words, price cutting is impossible as part of equilibrium behaviour, as emphasized, for example, by Friedman (1983 p. 228) – unless the price cutter expects its profits to increase even if its competitors follow suit.

available in a paper by d'Aspremont and Dos Santos Ferreira (2009), even if they note (on p. 81) that "the existence problem requires more development".

However, while sealed bidding may be enforced by a big buyer, as in construction or in sales to the public sector, sealed bidding is not applicable to markets where buyers take prices as given and prices are set by sellers, as in most consumer markets. Of course, in such markets prices are set by firms in the beginning of the market period, before production takes place, as in sealed bidding. But in contrast to sealed bidding firms can observe and revise their prices in the beginning of the market period and establish a market price at which all firms can trade and share the market, in contrast to sealed bidding where only the firm with the lowest bid will trade.

For markets where buyers take prices as given I therefore propose a model where firms choose prices cooperatively but production non-cooperatively. More precisely, the market price is set by a price leader and firms choose production non-cooperatively taking this market price as given. However, the price leader is not a *collusive* price leader, maximizing its industry's profits, but a *competitive* price leader, as defined in Section 3.

4.1 Choice of production by price-taking firms

Now, consider the determination of production at different market prices. Firms' outputs at a given market price are determined exclusively by supply only at the market-clearing price. At higher prices they will also depend on consumer choice (and *only* consumer choice, since we exclude the possibility for firms to fix market shares). If firms are identical, and no supply constraint is binding, the probability that a consumer chooses to buy from a particular firm will be $1/n$ if there are *n* firms (independent of the market price), and it follows from the law of large numbers that each firm's market share will be $1/n$.

However, firms try to increase their market shares by creating value for customers in ways which cannot easily be imitated by their competitors, suggesting that competition often is a *race* rather than a game. An obvious but difficult way of product differentiation is innovation, which gives the innovator a market share of 100 per cent to begin with. In general market shares depend on the phase of the product cycle (innovation, growth, maturity or decline) and the particular history of the market, as emphasized, for example, by Simon (1989) and Scherer (1996). This means that market shares are predetermined to a large extent.

But some marketing may also occur *during* the market period, when the market price has been set, particularly advertising and distribution, and such competition cannot be ignored, since it will affect the profits of each firm, including the price leader. When deriving its profits as a function of the market price, a price leader consequently has to anticipate the effects of marketing on profits. And to do this the price leader can use game theory.

To see what marketing implies for price leadership, define aggregate supply at the market price *p* in the usual way as $S(p) = \sum s_j(p)$, where the supply $s_j(p)$ of firm *j* is derived on the presumption that everything produced will be sold. This presumption is also true if $p = p^c$, where p^c clears the market, $D(p^c) = S(p^c)$, where $D(p)$ denotes demand at *p*. But it is not true if $p > p^c$. Producers should realize this and adjust production accordingly – assuming that they do take the market price *p* as given when production is determined.

At this stage market sharing has to be specified. Now, a firm's output can either influence its sales or not. If not, a firm will adjust its production to its sales, and market shares will be determined as in markets with production to orders as elaborated in Section 5, at least approximately.[8] On the other hand, market shares can also be influenced by making goods available to consumers in shops. And assuming that availability in shops is proportional to output distributed among shops in the market, a firm's market share will be

(1) $$\alpha_i = q_i \Big/ \sum q_j ,$$

where q_i denotes a firm's production. This is *proportional rationing*, where every unit of supply of a homogeneous good has the same probability of being sold in the market.

It follows that a firm's profit function is

(2) $$\pi_i = pD(p)\, q_i \Big/ \sum q_j - c_i(q_i),$$

where $c_i(\cdot)$ denotes a firm's cost function, assuming in addition (for simplicity) that output remaining at the end of the market period is without value.[9] Differentiation yields

(3) $$\frac{\partial \pi_i}{\partial q_i} = p(1-\alpha_i)\, d - c_i'(q_i),$$

where $d = D(p) \Big/ \sum q_j$. It follows that (q_i) is an equilibrium point if

(4) $$p(1-\alpha_i)\, d = c_i'(q_i) \text{ for every } i.$$

[8] Of course, sales can only be anticipated in a market where production precedes sales, and then sales will in general differ from production (and the difference will change the firm's inventories); hence the approximation.
[9] Note that stocks remaining at the end of the market period are often sold at a reduced price (or simply scrapped). In any case, adding an inventory evaluation function will not change the substance of the analysis.

Thus, a solution to these equations represents – if it is unique – rational predictions of firms' choices of production. But a solution is also self-enforcing in the sense that firms *ex post* have no reason to deviate from it (or regret it).

4.2 Price leadership in oligopolistic markets[10]

Next we consider an industry with *n* identical firms producing at constant returns with the marginal cost *c*. In this case (4) reduces to the system of equations $p(1-\alpha_i)d = c$, which is solved by $\alpha_i = 1/n$ and $d = (c/p)/(1-1/n)$ if $d \leq 1$ or, equivalently, $p \geq c/(1-1/n)$. Since $q_i = D(p)/nd$, firms taking the market price *p* as given will consequently in equilibrium restrict production to

(5) $$q_i^e(p) = (1-1/n)(p/c)D(p)/n \text{ if } p \geq c/(1-1/n).$$

In this case production is consequently not restricted by sales or capacity, as in markets where sales precede production. Instead it is endogenously determined by a marketing game.

Moreover, it follows from $\pi_i = (pd-c)q_i$, $d = (c/p)/(1-1/n)$ and (5) that a firm's profits in equilibrium after quantity adjustment at the market price *p* will be

(6) $$\pi_i^e(p) = pD(p)/n^2 \text{ if } p \geq c/(1-1/n).$$

If on the other hand $p < c/(1-1/n)$, then $q_i = D(p)/n$ is an equilibrium point, since for $\alpha_i = 1/n$ and $d = 1$ we have $\partial\pi_i/\partial q_i = p(1-\alpha_i)d - c = p(1-1/n) - c < 0$, implying that

(7) $$q_i^e(p) = D(p)/n \text{ and } \pi_i^e(p) = (p-c)D(p)/n \text{ if } p < c/(1-1/n).$$

It follows that every firm prefers $p^o = \arg\max pD(p)$ as the market price if demand is sufficiently inelastic, $p^o \geq c/(1-1/n)$, since $(p-c)D(p)$ is increasing in *p* up to p^m by assumption, $p^o < p^m$, and $\pi_i^e(p)$ is continuous at $p = c/(1-1/n)$. This completes the proof of the following result:

PROPOSITION 1. Consider a market where production precedes sales, rationing is proportional and there are *n* firms producing at constant returns with the same marginal cost *c*. Then all firms prefer the same market price, namely $p^o = \arg\max pD(p)$, if demand is sufficiently inelastic, $p^o \geq c/(1-1/n)$.

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[10] This subsection draws on Farm (1988), where, however, price leadership is not even mentioned.

Thus, costly competition for market shares through non-cooperatively chosen production will make it profitable for a price leader in an industry with identical firms to set a price which is lower than the monopoly price, namely that price which maximizes the industry's revenues. Note that, according to (6), new firms would reduce profits for incumbents not only at the rate of $1/n$, because of more firms sharing the same revenues, but at the rate of $1/n^2$, because of additional supply in equilibrium. Note also that the case of elastic demand is not particularly interesting, since then not even a monopolist would set a high price.

4.3 Price leadership in atomistic markets

We shall now see that the result above for identical firms – that they prefer a revenue-maximizing price instead of a monopoly price – can be generalized to atomistic markets, irrespective of the cost functions, provided that firms are so small that we can set $\alpha_i = 0$ in (4). For then it follows that $pd = c_i'(q_i)$ or, equivalently, $q_i = s_i(pd)$. Hence $\sum q_i = S(pd)$ and $d = D(p)/S(pd)$ so that $d = d(p)$ solves the equation

(8) $$D(p) = S(pd)d .$$

Assuming, as we always do in this paper, that $D(p)$ is decreasing in p and $S(p)$ increasing or constant, the solution to this equation is unique, and then we have the following result:

PROPOSITION 2. In a market where production precedes sales and rationing is proportional, 'small' firms taking a market price $p > p^c$ as given will in equilibrium produce

(9) $$q_i^e(p) = s_i(pd(p)),$$

where $d(p)$ is defined by (8).

Note that $d(p) < 1$ for $p > p^c$ so that (9) is indeed an interior solution (excluding market clearing) and $1 - d(p)$ is the equilibrium rate of excess supply.

Also note that $pD(p) = S(pd(p))pd(p)$, so that, with our assumptions on demand and supply, it follows from $(pD)' = S'(pd)(pd)' pd + S(pd)(pd)'$ that

(10) $$sign(pd(p))' = sign(pD(p))' .$$

Hence the equilibrium supply curve

(11) $$Q^e = Q^e(p) = S(pd(p))$$

is backward-bending at p if the usual potential supply curve $S(p)$ is forward-bending and demand is elastic, $-pD'(p)/D(p) = \eta(p) > 1$, since $(pD)' = D(1-\eta)$. If demand is elastic for every $p > p^c$, equilibrium supply will be less than $D(p^c)$ for every $p > p^c$. And if demand is inelastic at p^c, equilibrium supply will be increasing up to $p^o = \arg\max pD(p)$ and then decreasing.

The traditional supply curve $S(p)$ reflects potential output from every potential firm. The equilibrium supply curve $Q^e(p)$ reflects endogenous output restriction, including exit. A firm's output will be positive if and only if $s_i(pd(p)) > 0$.

Next we find that every firm prefers the same market price, irrespective of its cost function:

PROPOSITION 3. In a market where production precedes sales, rationing is proportional and firms are 'small', all firms prefer the same market, namely

(12) $$\max(p^c, p^o),$$

(13) $$\text{where } D(p^c) = S(p^c) \text{ and } p^o = \arg\max pD(p).$$

Proof. Recall that a firm's profit in equilibrium after quantity adjustment is

(14) $$\pi_i^e(p) = pd(p)q_i^e - c_i(q_i^e),$$

where $d(p)$ is defined above and $q_i^e = s_i(pd(p))$ maximizes $\pi_i = pd(p)q_i - c_i(q_i)$. It follows from the envelope theorem that

(15) $$\frac{d\pi_i^e(p)}{dp} = \frac{\partial(pd(p))}{\partial p} s_i(pd(p)),$$

and hence that $\pi_i^e(p)$ is maximized by $\arg\max pd(p)$, which is equal to $\arg\max pD(p)$ according to (10).

To complete the model it is hardly realistic in this case to assume that one of the small firms is a price leader. Instead we assume, as in the traditional model of perfect competition, that *all* firms are price takers. But since we here stick to the price-taking postulate, we also assume, in order to model an orderly market, that the industry has a trade association which sets the market price *but cannot restrict production*. And realizing that every firm prefers that market price which maximizes the industry's revenues, the trade association's problem is that of a statistician, namely to estimate the demand function and especially its price elasticity.

Is this model of a market with 'many' firms applicable to any real market? Grain farming is an industry with many firms, but then one can hardly assume that buyers take prices as given so that prices are completely determined by firms. (Instead prices are set in auction markets, including markets for futures, often also including intervention by a government.) But a taxi market is probably a relevant example. For instance, deregulation of a taxi market which introduces free entry will not lower the market price much but increase the number of cabs, according to the model above, so there will be some excess supply in equilibrium (if product demand is sufficiently inelastic).

This section has shown that equilibrium is possible in the usual sense (rational predictions, self-enforcing agreements) even with excess supply at a market price above the market-clearing level. The model predicts a market price which maximizes the industry's revenues in a market where buyers take prices as given, production precedes sales, and every unit of output has the same probability of being sold (proportional rationing). But the basic message of this bench-mark model is not that firms always manage to set a market price at which the price elasticity of product demand is *exactly* equal to 1. The main point is instead that firms sometimes, when product demand is sufficiently inelastic, choose to set a market price which does not clear the market, and that they adapt to this by restricting supply, even if some excess supply will remain. In practice the outcome will, of course, depend on the details of the market structure, including marketing and other deviations from proportional rationing. The outcome will also depend on firms' ability to estimate the price elasticity of product demand and their willingness to exploit it.

5. Pricing and production in markets where sales precede production

Next we study the implications of competitive price leadership in markets where sales precede production, so that firms only produce what they can sell. Carlton (1989 p. 941) expects "that our economy has increased its reliance on industries that produce to order", even if he "has not seen much research on this topic". Since production to orders eliminates costly excess supply, it may also appear profitable for all firms in an industry to introduce this system – whenever it is possible.

Services are, of course, always produced to order. Otherwise production to orders is possible whenever consumers can accept some waiting time between purchase and delivery. If consumers want to inspect a product before purchase, they will prefer shops where products are demonstrated, but they may accept some waiting time before a replica of the product is delivered from the factory, implying production to orders.

For simplicity we always assume in this section that a firm's marginal cost is constant up to a certain level of production – its capacity – where it becomes so strongly increasing that its potential output can be approximated by its capacity even for high prices. A firm's marginal cost function is then characterized by two parameters: its unit cost and its capacity. This is probably not only a useful first approximation but also rather realistic.

Consider first an industry with *n* identical firms facing the same market price. Then every firm has the same probability of being contacted by a consumer, so market shares must be equal (according to the law of large numbers). In a market where sales determine production the profit of a firm, taking a market price $p \geq c$ as given, is then equal to $\left(p-c\right)D\left(p\right)/n - rk$ if $p \geq p^k$, and $\left(p-c-r\right)k$ if $p \leq p^k$, where *c* denotes unit cost, *k* a firm's capacity, rk its fixed costs, and $D\left(p^k\right) = nk$, where *D* is the demand function.

The traditional model of perfect competition predicts the market-clearing price p^k as the market price if $p^k > c$. If $p^k \leq c$ it predicts marginal-cost pricing and negative profits until the shutdown of some of the firms establishes a capacity-clearing price which yields non-negative profits for the remaining firms. But all firms prefer $\max\left(p^m, p^k\right)$ as the market price, where p^m maximizes $\left(p-c\right)D\left(p\right)$, so that if we complete the model with a price leader, the market price will cover not only variable costs but also at least some fixed costs if $p^k \leq c$.

On the other hand, even a price leader will set the market-clearing price if $p^k > p^m$. In other words, market-clearing occurs if firms are producing at full capacity and a higher market price would reduce profits for every firm. But this is a special case. In general the market price will be market-clearing if and only if a higher market price would reduce profits for at least one firm. It remains to see how marketing, cost differentials, capacity differentials, and different market shares can affect prices.

5.1 Effects of marketing

In markets with production to orders, a firm's output does not determine but is determined by its market share. However, a firm can influence its market share by other means than output, and we call such measures marketing. Following Shubik with Levitan (1980 p. 192-194), we assume that a firm's market share is

(16) $$\beta_i = (1-\gamma)\alpha_i + \gamma\, a_i \big/ \sum a_j\ ,\ 0 < \gamma \le 1,$$

where α_i denotes its market share in the absence of marketing, a_i denotes the firm's expenditures on marketing and γ measures the effect of this marketing, assumed (for simplicity) to be the same for every firm.

Shubik with Levitan interprets a_i as expenditures on advertising and γ as the proportion of customers who are influenced by advertising, but other interpretations are possible, like distribution of goods to shops, provided they only include measures which are made and have effects during the market period. Assuming that there are *n* firms, the market shares α_i may be equal to $1/n$ or determined by previous marketing expenditures.

Now, when deriving profits as a function of the market price, a firm has to anticipate the effects of marketing on profits. And with marketing technology according to (16) a firm's profit (excluding fixed costs) in a market with demand function *D* and market price *p* is

(17) $$\pi_i = (p-c)D(p)\left[(1-\gamma)\alpha_i + \gamma\, a_i \big/ \sum a_j\right] - a_i\,,$$

assuming for simplicity that all firms have the same unit cost (*c*). It follows that

(18) $$\partial\pi_i / \partial a_i = (p-c)D(p)\gamma\frac{1-a_i/A}{A} - 1,$$

where $A = \sum a_j$, so that in equilibrium at $p > c$,

(19) $$a_i / A = 1/n\,,$$

(20) $$A = (p-c)D(p)\gamma(1-1/n)\,,$$

(21) $$\pi_i = (p-c)D(p)\left[(1-\gamma)\alpha_i + \gamma/n^2\right].$$

Marketing will consequently affect profits but not preferred prices in equilibrium.

However, introducing capacity constraints, and assuming for simplicity that all firms have the same capacity (*k*) and the same α_i, a firm's profits as a function of the market price *p* (in marketing equilibrium) will be

(22) $$\pi_i(p) = (p-c)D(p)/m \ \text{ if } \ p > p^k\,,$$

(23) $$\pi_i(p) = (p-c)K/n \text{ if } p \le p^k,$$

where $D(p^k) = K = nk$ and

(24) $$1/m = (1-\gamma)/n + \gamma/n^2,$$

assuming that marketing when $p \le p^k$ can be ignored. Note that $\pi_i(p)$ is discontinuous at $p = p^k$, since $1/m < 1/n$ when $\gamma > 0$ and $n > 1$. Since $(p-c)D(p)$ is increasing in *p* up to p^m by assumption we then have the following result:

PROPOSITION 4. Consider *n* firms with the same constant marginal cost *c* up to capacity and the same capacity (K/n) in a market with production to orders and marketing according to (16) with $\alpha_i = 1/n$. Then a price leader will set

(25) $$p^m \text{ if } K > K_d \text{ and } p^k \text{ if } K < K_d,$$

where $p^m = \arg\max(p-c)D(p)$, $p^k = P(K)$, and K_d is determined by the equation

(26) $$\left(P(K_d) - c\right)K_d = \left((1-\gamma) + \gamma/n\right)\left(p^m - c\right)D\left(p^m\right),$$

where *P* denotes the inverse of the demand function *D*.

Note that, in this case, a price leader will set a capacity-clearing price p^k not only for $K \le D(p^m)$, as in a price leader model with exogenous market shares. Instead we have capacity clearing and a market price below p^m as long as $K \le K_d$, with K_d even approaching $D(c)$ as $n \to \infty$ if $\gamma = 1$. The threat of costly competition for market shares in excess-capacity situations will enforce capacity clearing provided that $K \le K_d$, so that the profit guarantee at capacity clearing is sufficiently high.

5.2 Effects of different costs

So far we have assumed identical firms. But in most industries firms are different, either for historical reasons or because of investments in attempts to raise market shares or reduce costs. Let us first briefly note the effect of cost differentials. Suppose there is one low-cost firm in the market with marginal cost c_1, and let p_1^m maximize $(p-c_1)D(p)$. Taking the market price as given by p_1^m, it may be tempting for a high-cost firm (c_n) to enter the market, provided that $c_n < p_1^m$. But the market share and the profits of the low-cost producer will decline as high-cost firms enter the market. And then, assuming that each firm captures an equal share of the market, the low-cost firm will prefer c_n instead of p_1^m as the market price if

(27) $$\left(c_n - c_1\right) D\left(c_n\right) > \left(p_1^m - c_1\right) D\left(p_1^m\right) / n,$$

i.e., if the number of firms (*n*) is so large that the profits at a low price (c_n) and a big market share (1) is higher than the profits at a high price (p_1^m) and a small market share ($1/n$). And assuming that $\left(p - c_1\right) D\left(p\right)$ is increasing in *p* up to p_1^m, we have the following simple explanation of price competition by low-cost producers:

PROPOSITION 5. Consider one low-cost firm (with unit cost c_1) and $n-1$ high-cost firms (with unit costs c_n) in a market with production to orders and constant returns, and suppose that $c_n < p_1^m$ where p_1^m maximizes $\left(p - c_1\right) D\left(p\right)$. Then the low-cost firm will set

(28) p_1^m if $c_n \leq c_s$ and c_n if $c_n > c_s$, where c_s is defined by

(29) $$\left(c_s - c_1\right) D\left(c_s\right) = \left(p_1^m - c_1\right) D\left(p_1^m\right) / n.$$

Thus, threat of entry of high-cost firms may force a low-cost firm to marginal cost pricing with respect to the high cost if the high cost is not too low, $c_n > c_s$. Note that here the price leader cuts its price in order to *eliminate* high-cost competitors ('cut-throat' competition). In the next subsection, with different capacity constraints, a price leader may find a price-cut profitable even if it does not eliminate other firms.

5.3 Effects of different capacities

Since price leadership by a big firm dominated the early literature, including Markham (1951), this form of price leadership deserves special attention. This case is also related to the classical notion of the competitive effect of many small firms, because it shows how the entry of many small firms can force a monopolist to lower its price. However, to show this we cannot simply *assume* that small firms can sell what they like to produce at every price set by the big firm, as the literature has done so far. In fact, as we shall see below, this assumption relies on an implicit assumption of market sharing at high prices which is too strong. Introducing reasonable assumptions on market sharing we shall see when and why the dominant firm finds it profitable to set a low price.

Consider first a monopoly with unit cost *c* and capacity k_n and suppose (for simplicity) that the demand function $D(\cdot)$ is linear, with

(30) $$D\left(p^e\right)=0.$$

Then the monopolist will maximize profits by setting its price equal to $\max\left(p^m, p^k\right)$, where p^k is the capacity-clearing price, $D\left(p^k\right)=k_n$, and

(31) $$p^m = c+\left(p^e-c\right)/2.$$

Next we introduce a 'competitive fringe' consisting of ν small firms, each with capacity $k_1 < k_n$ and unit cost *c*. Suppose first that these firms can sell what they like to produce at every price set by the big firm, as in Scherer (1980 p. 232). Then the big firm's residual demand curve is given by

(32) $$D_n\left(p\right)=D\left(p\right)-\nu k_1,\ c\le p\le p^f \text{, where } D\left(p^f\right)=\nu k_1 \text{ and } D_n\left(p^f\right)=0.$$

When the demand curve is linear, the residual demand curve is obviously also linear, and it follows from (30) and (31) that the big firm's profit-maximizing price is $\max\left(p^o, p^k\right)$, where p^k now is defined by $D\left(p^k\right)=\nu k_1+k_n$, and

(33) $$p^o = c+\left(p^f-c\right)/2.$$

It is easy to show (see Appendix 2) that

(34) $$D\left(p^o\right)=D\left(p^m\right)+\nu k_1/2,$$

which implies that

(35) $$p^k < p^o \text{ if and only if } k_n+\nu k_1/2 > D\left(p^m\right),$$

so that

(36) $$\max\left(p^o, p^k\right)=p^o \text{ if } k_n \ge D\left(p^m\right),$$

which we assume for simplicity in what follows. Note that p^o is indeed decreasing in the total capacity of the fringe firms (νk_1) and that we even have $p^o \to c$ as $\nu k_1 \to D(c)$. It is also easy to show (see Appendix 2) that

(37) $$\left(p^o-c\right)\left(D\left(p^o\right)-\nu k_1\right)=\left(p^m-c\right)D\left(p^m\right)\left(1-\nu k_1/D(c)\right)^2.$$

Now, this traditional model of the competitive effect on a dominant firm from a fringe of small firms relies on the assumption that small firms can sell what they like to produce at every price set by the big firm, including $p = p^f$. But at this price the market share of the small firms is 1 while the market share of the dominant firm is 0, which is a very extreme assumption on market sharing.

Suppose instead that every firm's production is restricted by sales and not capacity for market prices greater than the capacity-clearing price p^k. This means that $\alpha_i D(p) < k_i$ for every *i* and every *p* greater than p^k, where α_i denotes a firm's market share when no capacity constraint is binding. And this is true if $\alpha_i = k_i / \sum k_j$, which may happen, for example, if investment in outlets has been adjusted to capacities, so that a firm's market share is proportional to its capacity even when no capacity constraint is binding and not only when the market price is capacity-clearing. In this case every firm, including the big one, prefers p^m as market price when $p^k < p^m$.

On the other hand, if products are perfect substitutes in every sense, including availability to consumers, then every firm's market share will be $1/n$ when no capacity constraint is binding. And this happens if $D(p)/n < k_i$ for every *i*, or, equivalently, if $p > D^{-1}(nk_1)$. In this case the big firm's demand function is:

(38) $$D_n(p) = D(p)/n \text{ if } p \geq p^u \text{, where } p^u = D^{-1}(nk_1),$$

(39) $$D_n(p) = D(p) - \nu k_1 \text{ if } p \leq p^u.$$

Thus, if nk_1 is so large that $p^u \leq p^m$ the big firm will prefer p^m if

(40) $$(p^o - c)(D(p^o) - \nu k_1) < (p^m - c) D(p^m)/n,$$

which, substituting (37), is equivalent to

(41) $$\nu k_1 / D(c) > 1 - 1/\sqrt{n},$$

suggesting the following result (with a complete proof in Appendix 2):

PROPOSITION 6. Consider ν small firms with capacity k_1 and one big firm with capacity k_n producing with unit cost *c* in a market with production to orders. Suppose that all firms have the same market share when no capacity constraint is binding. Also assume that the demand function $D(\cdot)$ is linear and that $k_n \geq D(p^m)$, where $p^m = \arg\max (p-c) D(p)$. Then the market price preferred by the big firm is:

(42) $$p^o \text{ if } \nu k_1 \leq D(c)\left(1 - 1/\sqrt{n}\right),$$

(43) $$p^m \text{ if } \nu k_1 \geq D(c)\left(1-1/\sqrt{n}\right),$$

(44) $$\text{where } n=\nu+1,\ p^m=c+\left(p^e-c\right)/2 \text{ and } p^o=c+\left(p^f-c\right)/2,$$

(45) $$\text{where } D\left(p^e\right)=0 \text{ and } D\left(p^f\right)=\nu k_1.$$

Thus, the market price set by a dominant price leader can indeed be reduced by the presence of small firms, and then it is also decreasing in the total capacity of the small firms. But now we also see that this is true only if the total capacity of the small firms is not too large. Moreover, since some calculation (in Appendix 2) shows that

(46) $$p^o=c+\left(p^m-c\right)\left(1-\nu k_1/D(c)\right),$$

we also have

(47) $$p^o \geq c+\left(p^m-c\right)/\sqrt{n} \text{ if } \nu k_1 \leq D(c)\left(1-1/\sqrt{n}\right),$$

so that the price set by a dominant firm can approach marginal cost pricing only if the small firms are 'sufficiently many' at the same time as their total capacity is 'sufficiently small'.

5.4 Effects of different market shares

Firms with the same capacity may prefer different market prices if they have different market shares, as argued already by Boulding (1941 p. 612) for the retail gasoline industry, where firms may have (approximately) the same capacity but different market shares due to different locations. And then a firm with a small market share may find price cutting profitable even if its competitors follow suit.

To see this, consider two firms in a market with the demand function *D*, each with unit cost *c* and capacity *k* and market shares α_1 and α_2 if no capacity constraint is binding, $\alpha_i D(p) \leq k$ for every *i*, which happens if $\alpha_2 D(p) \leq k$, assuming that $\alpha_1 < \alpha_2$. It follows that (the variable part of) the profits of the firm with the larger market share as a function of the market price is

(48) $$\pi_2(p)=(p-c)\alpha_2 D(p) \text{ if } p \geq p^u, \text{ where } p^u=D^{-1}(k/\alpha_2),$$

(49) $$\pi_2(p)=(p-c)k \text{ if } p \leq p^u,$$

while the profits of the firm with the smaller market share is

(50) $$\pi_1(p)=(p-c)\alpha_1 D(p) \text{ if } p \geq p^u,$$

(51) $$\pi_1(p)=(p-c)\left(D(p)-k\right) \text{ if } p^k \leq p \leq p^u, \text{ where } p^k=D^{-1}(2k),$$

(52) $$\pi_1(p) = (p - c)k \text{ if } p \le p^k.$$

Thus it is only the firm with the small market share which can increase its market share and sometimes also its profits by reducing the market price (when $p^k \le p \le p^u$). In fact, as proved in Appendix 2, we have the following result:

PROPOSITION 7. Consider two firms, each with capacity *k* and unit cost *c* in a market with production to orders. Let their market shares be α_1 and α_2 if no capacity constraint is binding and suppose that $\alpha_1 < \alpha_2$. Also assume (for simplicity) that the demand function $D(\cdot)$ is linear and that $k \ge D(p^m)$, where $p^m = \arg\max(p-c)D(p)$. Then the market price preferred by the firm with the smaller market share is:

(53) $$p^o \text{ if } \alpha_1 < \left(1 - k/2D(p^m)\right)^2,$$

(54) $$p^m \text{ if } \alpha_1 > \left(1 - k/2D(p^m)\right)^2,$$

(55) $$\text{where } p^m = c + (p^e - c)/2 \text{ and } p^o = c + (p^f - c)/2,$$

(56) $$\text{where } D(p^e) = 0 \text{ and } D(p^f) = k.$$

6. Conclusions

This paper considers markets where consumers take prices as given and price setting is decentralized to producers. In such markets there is neither an auctioneer enforcing market-clearing nor a big buyer enforcing sealed bidding. Instead there is, as I have argued in Section 3, a price leader. More precisely the market price is set by a firm preferring the lowest market price, while all other firms take this price as given. This means that the market price goes down if and only if a price cut is profitable for a firm *even if its competitors follow suit*, while the market price goes up if and only if a higher market price is profitable for every firm.

I have argued that the outcome of this market form, which may be called *competitive price leadership*, crucially depends on the way the market is shared between firms at different market prices. And the distribution of sales between firms depends in general not only on firms' capacities, as it does at the market-clearing price, but also on consumers' preferences, marketing (design, advertising and distribution), and whether sales precede production or not.

Competitive price leadership means that pricing is only partially cooperative. If a firm by price cutting can increase its market share so much that its profits also increase even if its competitors follow suit, then it will also cut its price. Price leadership by a dominant firm is a classical example, and we have seen in detail when and why the presence of many small firms will force a big firm to abandon monopoly pricing. Thus, competitive price leadership does not exclude price competition, only price competition which reduces profits for every firm.

The market price may be reduced below the monopoly level even when all firms prefer the same market price, namely if there is competition in other variables than prices. For example, in markets where production precedes sales and every unit of output has the same probability of being sold, all firms will prefer the same market price, at least in industries with identical or very small firms, and this market price will maximize the industry's sales revenues. Another example, in markets where sales precede production, is that the threat of costly competition for market shares in situations with excess capacity may reduce the market price to the market-clearing level even for identical firms.

Of course, market clearing will also be established by competitive price leadership if all firms in an industry are producing at full capacity and a higher market price would reduce profits for at least one firm. This is a simple and plausible explanation for market clearing in a boom, but competitive price leadership also explains pricing when production is restricted by sales and not capacity during a recession.

Appendix 1. Relation to the management literature

This appendix relates competitive price leadership to pricing as discussed in the management literature, that is, cost-plus pricing and value-based pricing. Both methods ignore the possibility of market clearing, with price determined by equality between demand and supply (capacity) in markets with production to orders. This means that both methods implicitly assume that a firm's production is restricted by sales (and not capacity), suggesting that this is a stylized fact for most firms most of the time.

Both cost-plus pricing and value-based pricing determine prices by mark-ups on unit costs, but the first method has a mark-up determined by fixed costs and estimated sales, while the mark-up is determined by estimated price elasticity of the firm's product demand with the second method. And while both methods can explain initial price revisions in the beginning of a market period, we shall see that they cannot always explain the formation of a market price.

Cost-plus pricing

Cost-plus pricing, or, synonymously, full-cost or average-cost pricing, means that firms set prices to cover all costs, including fixed costs, where the contribution of fixed costs to the price is obtained by dividing fixed costs for the market period by variable costs of estimated sales. More precisely, and assuming that a firm's marginal cost is constant (c) up to a certain fixed capacity (k), the product price is determined by

(1) $$p = c + mc \text{, where } m = rk/cq \text{,}$$

where q denotes estimated sales, $q \leq k$, and rk capital costs (including normal profits). Thus, cost-plus pricing implies that fixed ('indirect') costs are allocated to a firm's products in proportion to variable ('direct') costs. And cost-plus pricing is a common pricing procedure in a market economy.[11]

A basic problem with cost-plus pricing is, of course, that a mark-up according to full-cost pricing is not necessarily profit-maximizing, not even for a monopolist, since the profit-maximizing price is independent of fixed costs. However, as long as actual sales (s) are at least equal to estimated sales (q), profits are at least normal (since profits equal $mcs = rks/q$), and then it may be rational for a firm to be satisfied with cost-plus pricing – if further information on demand is too costly. A lower price may appear too risky because the firm does not know if demand is sufficiently elastic, and a higher price may also appear too risky because the firm does not know if demand is sufficiently inelastic. After all, estimating

[11] See e.g. Okun (1981 p. 153), Simon (1989 p. 48) and Nagle and Hogan (2006 p. 2).

product demand, and in particular the price elasticity of product demand, is a difficult problem, particularly for a firm with many products.

Moreover, revenues should cover all costs, including fixed costs, since there is no point in profit maximization unless the resulting profit is positive. Thus, cost-plus pricing can be interpreted as the first stage in a two-stage process. If cost-plus prices give positive profits the firm knows that there are at least some prices which make its production profitable. And in the second stage the firm may attempt to optimize prices, particularly in a recession, when raising prices as required by cost-plus pricing may ruin the firm.

Cost-plus pricing has also been interpreted as “tacit collusion”,[12] in spite of the fact that cost-plus prices are not necessarily profit-maximizing. Perhaps cost-plus pricing should be interpreted instead as ‘tacit coordination’, but only if firms have similar cost structures. If full costs differ so much that the corresponding price differentials cannot be sustained, it remains to explain why and how prices are adjusted. Thus, cost-plus pricing may explain initial price revisions but not always the formation of a market price.

Value-based pricing

Moreover, cost-plus pricing can even be dismissed as a “delusion” in modern management literature, leading to “overpricing in weak markets and underpricing in strong markets” (Nagle and Hogan 2006 p. 3). What is advocated instead is pricing based on “how products and services create value for customers” (Nagle and Hogan 2006 p. 27) or, in other words, profit maximization.

To recall what profit-maximization implies for pricing, assume that a firm only produces what it can sell (produce to order). Then the firm’s profit as a function of its price is $\pi = pq - C$, where sales depends on price, $q = D(p)$, and costs depends on production, $C = C(q)$. Hence $d\pi/dp = q(1-\mu\eta)$, where μ is the contribution margin, $\mu = (p - C')/p$, with C' denoting marginal cost, $C' = C'(q)$ and η the price elasticity of sales, $\eta = -(dq/q)/(dp/p)$. The profit-maximizing price is determined implicitly by

(2) $$\mu = 1/\eta .$$

This equation is often used to guide price management in practice (see e.g. Simon 1989). If, for example, the contribution margin μ is 50 %, then this margin is also profit-maximizing

[12] See e.g Stigler (1947 p. 433) and Simon (1989 p. 79).

if changing price by 1 % is estimated to change sales by 2 %. And if the firm's marginal cost is constant, $C'(q)=c$, up to its capacity *k*, the product price is determined by

(3) $$p^m = (1+m)c \text{, where } m = 1/(\eta - 1),$$

as long as $D(p^m) \le k$. Thus, while cost-plus pricing has the same mark-up for all products, value-based pricing implies mark-ups which are inversely proportional to the products' price elasticities (minus 1).

However, this form of the mark-up only applies to a monopoly or a small firm assuming that its price revisions will affect only its customers and not its competitors, and that the influence of other prices on its sales can be neglected. If value-based prices differ so much between firms that the corresponding price differentials cannot be sustained, it remains to explain why and how prices are adjusted. In other words, not even value-based pricing can always explain the formation of a market price.

For an individual firm the problem is whether it is forced to adjust to another firm's price or not. If all firms prefer the same market price, no adjustment will be needed, provided all firms begin by announcing their preferred market prices. And if price preferences differ, firms should adjust to the lowest price. This pricing, which is equivalent to competitive price leadership, may alternatively be called *market-based pricing*. It is based on individual market shares instead of individual demand functions, in addition to the usual market demand function. It also determines whether a firm should be a price maker or a price taker.

Appendix 2. Proofs

Consider first a linear demand function $D(\cdot)$ and define p^e and p^f by

(1) $$D(p^e) = 0 \text{ and } D(p^f) = k.$$

Moreover, define p^m and p^o by

(2) $$p^m = \arg\max (p-c)D(p) \text{ and } p^o = \arg\max (p-c)(D(p)-k),$$

and recall that

(3) $$p^m = c + (p^e - c)/2,\ D(c) = 2D(p^m), \text{ and } p^o = c + (p^f - c)/2.$$

Note that our linear demand function can now be written as

(4) $$D(p) = D(c)\left(1 - \frac{p-c}{2(p^m - c)}\right).$$

Substituting p^f in (4) we obtain

(5) $$k = D\left(p^f\right) = D(c)\left(1 - \frac{p^f - c}{2\left(p^m - c\right)}\right) = D(c)\left(1 - \frac{2\left(p^o - c\right)}{2\left(p^m - c\right)}\right),$$

and hence also

(6) $$p^o - c = \left(p^m - c\right)\left(1 - k/D(c)\right).$$

Substituting p^o in (4) and using (6) and (3) we obtain

(7) $$D\left(p^o\right) = D(c)\left(1 - \frac{p^o - c}{2\left(p^m - c\right)}\right) = D(c)\left(1 - \frac{1}{2}\left(1 - \frac{k}{D(c)}\right)\right) = D\left(p^m\right) + k/2.$$

It follows from (6) and (7) that

(8) $$\left(p^o - c\right)\left(D\left(p^o\right) - k\right) = \left(p^m - c\right)\left(1 - k/D(c)\right)\left(D\left(p^m\right) - k/2\right),$$

and combining this with $D(c) = 2D\left(p^m\right)$ we find that

(9) $$\left(p^o - c\right)\left(D\left(p^o\right) - k\right) = \left(p^m - c\right)D\left(p^m\right)\left(1 - k/D(c)\right)^2.$$

Proof of proposition 6

We want to prove that

(10) $$p_n^p = p^o \text{ if } \nu k_1 < D(c)\left(1 - 1/\sqrt{n}\right), \text{ and}$$

(11) $$p_n^p = p^m \text{ if } \nu k_1 > D(c)\left(1 - 1/\sqrt{n}\right),$$

where p_n^p denotes the market price preferred by the big firm,

(12) $$p_n^p = \arg\max \pi_n(p) = (p - c)D_n(p), \text{ where}$$

(13) $$D_n(p) = D(p)/n \text{ if } p \geq p^u, \text{ where } p^u = D^{-1}\left(nk_1\right),$$

(14) $$D_n(p) = D(p) - \nu k_1 \text{ if } p \leq p^u.$$

To do this we begin by defining

(15) $$\pi\left(p^o\right) = \left(p^o - c\right)\left(D\left(p^o\right) - \nu k_1\right),$$

(16) $$\pi\left(p^m\right) = \left(p^m - c\right)D\left(p^m\right)/n,$$

and note that, according to (9) with $k = \nu k_1$ and $D(c) = 2D\left(p^m\right)$,

(17) $$\pi\left(p^o\right) > \pi\left(p^m\right) \text{ if } \nu k_1/2D\left(p^m\right) < 1 - 1/\sqrt{n}, \text{ and}$$

(18) $$\pi\left(p^m\right) > \pi\left(p^o\right) \text{ if } \nu k_1 / 2D\left(p^m\right) > 1 - 1/\sqrt{n}.$$

Now, to prove (11) it is sufficient to prove that $p^u < p^m$ if $\nu k_1 / 2D\left(p^m\right) > 1 - 1/\sqrt{n}$, since then $\pi_n\left(p^m\right) = \pi\left(p^m\right) > \pi\left(p^o\right)$ and $\pi\left(p^o\right) \geq \pi_n\left(p\right)$ for every $p \leq p^u$. And if $k_1 / D\left(p^m\right) > 2\left(1 - 1/\sqrt{n}\right)/\left(n-1\right)$ we do have $k_1 / D\left(p^m\right) > 1/n$ and $p^u < p^m$, since $1/n < 2\left(1 - 1/\sqrt{n}\right)/\left(n-1\right)$ and this inequality is equivalent to $0 < \left(n-1\right)^2$, which is true for every $n > 1$.

To prove (10), suppose first that $p^u \geq p^m$ or, equivalently, $nk_1 \leq D\left(p^m\right)$. Then $\pi_n\left(p\right)$ is decreasing in p above p^u so that $p_n^p \leq p^u$. But to prove that $p_n^p = p^o$ we also have to prove that $p^o \leq p^u$ or, equivalently, $nk_1 \leq D\left(p^o\right)$. And substituting $D\left(p^o\right) = D\left(p^m\right) + \left(n-1\right)k_1/2$ we find that

(19) $$p^o \leq p^u \text{ if and only if } \left(n+1\right)k_1 \leq 2D\left(p^m\right).$$

Moreover, if $nk_1 \leq D\left(p^m\right)$ then $k_1\left(n+1\right)/2 < D\left(p^m\right)$, since $\left(n+1\right)/2 < n$ if $n > 1$, and it follows from (19) that $p^o \leq p^u$. Finally, to prove (10) when $p^u < p^m$, note first that in this case (10) follows from (17) if $p^o \leq p^u$. And $p^o \leq p^u$ follows from (19) and $k_1 / 2D\left(p^m\right) < \left(1 - 1/\sqrt{n}\right)/\left(n-1\right)$ since $\left(1 - 1/\sqrt{n}\right)/\left(n-1\right) < 1/\left(1+n\right)$ and this inequality is equivalent to $0 < \left(n-1\right)^2$, which is true for every $n > 1$.

Proof of proposition 7

Let p_1^p denote the market price preferred by firm 1 and define

(20) $$\pi\left(p^o\right) = \left(p^o - c\right)\left(D\left(p^o\right) - k\right) \text{ and } \pi\left(p^m\right) = \alpha_1\left(p^m - c\right)D\left(p^m\right).$$

From the assumption that $k \geq D\left(p^m\right)$ it follows that $p^u < p^m$, where $p^u = D^{-1}\left(k/\alpha_2\right)$. Hence $p_1^p = p^o$ if $\pi\left(p^o\right) > \pi\left(p^m\right)$ and $p^o < p^u$. Moreover, $p^o < p^u$ if $D\left(p^o\right) > D\left(p^u\right)$ which according to (7) is equivalent to $D\left(p^m\right) + k/2 > k/\alpha_2$. Substituting $\alpha_2 = 1 - \alpha_1$ it follows that

(21) $$p^o < p^u \text{ if } \alpha_1 < \frac{1 - k/2D\left(p^m\right)}{1 + k/2D\left(p^m\right)}.$$

Hence $p_1^p = p^o$ if $\alpha_1 < \left(1 - k/2D\left(p^m\right)\right)^2$, since then we have both $\pi\left(p^o\right) > \pi\left(p^m\right)$ according to (9) and $p^o < p^u$ according to (21), since

(22) $$\left(1 - k/2D\left(p^m\right)\right)^2 < \frac{1 - k/2D\left(p^m\right)}{1 + k/2D\left(p^m\right)}.$$

Finally, if $\alpha_1 > \left(1 - k/2D\left(p^m\right)\right)^2$ then $\pi\left(p^o\right) < \pi\left(p^m\right)$, and since $p^u < p^m$ we also have $\pi_1\left(p^m\right) > \pi_1\left(p\right)$ for every $p \le p^u$.

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