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# LABOUR DEMAND AND REAL WAGES

by

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*Abstract:* This paper shows that the traditional model of labour demand in a firm is incomplete and that the determinants of labour demand in a complete model include the demand for the firm's products but not the real wage.

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#### **1. Introduction**

What are the determinants of employment in the short run, when the capital stock is given? The most well-known proposition in this field is that a competitive firm chooses its level of employment by setting the value of the marginal product equal to the wage,

(1) 
$$pF'(N) = w \text{ if } F'' < 0$$

where N denotes employment (in hours), w the wage, p the product price, and F the production function. And the corresponding result for a non-competitive firm is

(2) 
$$(1-1/\eta)pF'(N) = w \text{ if } F'' < 0$$

where  $\eta$  denotes the price elasticity of the firm's product demand.<sup>1</sup>

Both (1) and (2) suggest that labour demand only indirectly depends on the level of product demand, which makes it difficult to explain the transmission of product-demand shocks to the labour market, as emphasized, for example, by Lindbeck (1998). Of course, (2) suggests that employment depends on product demand through its price elasticity, as also noted by Hamermesh (1993 p. 22), but since  $p = D^{-1}(F(N))$  the relation between product demand and employment is not particularly transparent. And according to (1) the labour demand of a competitive firm does not depend on product demand at all, only on the production function and the real wage (w/p).

On the other hand, as emphasized in (almost) every textbook in economics, labour demand is a derived demand. The dependence of labour demand on product demand through a price related to costs is also emphasized in most textbooks *at the industry level*, for example in Borjas (2008 p. 131), when Marshall's laws of derived demand are discussed. And if labour demand is a derived demand at the industry level, it should also be a derived demand at the firm level. The purpose of this paper is to show that this is indeed the case – in markets where prices are set by firms and not by a market-clearing auctioneer or process.

The intuition is as follows. The traditional motivation for (1) and (2) is that a profitmaximizing firm increases employment until the value of the marginal product of labour is equal to the wage rate. However, in practice a firm does not chose employment but the price of its output before trade can start. And then it adjusts its production to its sales and its employment to its production. Thus, a firm's employment must depend on the demand for its products at the prices it sets.

<sup>&</sup>lt;sup>1</sup> See any introductory textbook in economics or labour economics but also, for instance, Hamermesh (1993 p. 22) and Cahuc and Zylberberg (2004 p. 175).

To model this formally we begin by looking at a non-competitive firm in Section 2, followed by a reappraisal of the traditional approach to labour demand for a competitive firm in Section 3. In both cases we find that the traditional model is incomplete and that complete models include product demand as one of the determinants of labour demand.

The traditional model is incomplete because it excludes the case when F'' = 0. In this case the marginal productivity is constant, F'(N) = a, and it is particularly clear that (1) and (2) must be interpreted as price equations. The fundamental problem with (1) and (2) is not that they are false but that they only indirectly affects employment, through effects on the product price, while the direct determinants include product demand and production function, as we shall see in the following two sections. The resultant model of a firm's employment is then generalized to include intermediate goods in Section 4 and recruitment costs in Section 5, while Section 6 deals with aggregation and Section 7 concludes.

Throughout the paper we study labour demand as usually defined, i.e. the relation between employment and its determinants on the assumption that firms can hire all the labour they want. We also assume that firms are free to adjust prices and employment to wages once wages have been set, with or without bargaining.

#### 2. Labour demand of a non-competitive firm

As emphasized, for example, by Layard, Nickell and Jackman (1991 p. 341), eq. (2) is an equilibrium relationship: "It is not a labour demand function because prices are chosen jointly with employment". Thus, in a market with monopoly or monopolistic competition, and assuming only one input to begin with,<sup>2</sup> a firm's problem is

(3) 
$$\max_{p,N} pF(N) - wN \text{ s.t. } F(N) = D(p),$$

where D(p) denotes the firm's sales at the price p. The traditional way of solving this problem is to substitute  $p = D^{-1}(F(N))$  and differentiate with respect to N. But this is an approach which does not correspond to problem (3). Instead it (implicitly) presupposes that the firm determines employment (and output), while the product price is set by a marketclearing auctioneer or process once output has been set.

However, in markets where buyers take prices as given, as in most consumer markets, trade cannot start until prices have been set by the firm. Production and employment are then

 $<sup>^{2}</sup>$  A model with labour as the only input is not so unrealistic as it seems, since *w* can be interpreted as total variable costs per hour, not only labour costs but also costs for intermediate goods used in the production process, as elaborated in Section 4.

adjusted by the firm to the sales determined by the firm's customers – as long as production is restricted by sales and not by capacity.

Of course, production does not adjust perfectly to sales unless sales precede production (production to orders). In markets where production precedes sale, as in most consumer markets, production will in general differ from sales, but then we can assume, as a first approximation, that the change in inventories is negligible.

To see what the firm's decision problem is in more detail, we begin by rewriting (2) as

$$(4) p = \mu w / F'(N)$$

where  $\mu = 1/(1-1/\eta)$ . This equation is valid even when the marginal product of labour is constant, F'(N) = a. In this case it is also easy to see that *p* is determined by the equation (5)  $(p-c)/p = 1/\eta(p)$ , where c = w/a.

If for example, the contribution margin 
$$(p-c)/p$$
 is 50%, then this margin

If, for example, the contribution margin (p-c)/p is 50%, then this margin is also profit maximizing if changing price by 1% is estimated to change sales by 2%.

Thus, it is clear that (2) should be interpreted not as a labour-demand function but as a *price equation* – as also emphasized in macroeconomic literature based on the price-setting curve (e.g. Layard et al. 1991). However, what I want to emphasize here is the determination of employment at the firm level. And employment in a firm is determined by

(6) 
$$N = (1/a) D(\mu w/a)$$
 if  $F'(N) = a$ .

In general, when  $F'' \le 0$ , problem (3) is solved by the three equations  $F(N) + \lambda D'(p) = 0$  $pF'(N) - w - \lambda F'(N) = 0$  and F(N) = D(p), where  $\lambda$  is the Lagrange multiplier. Hence

$$\lambda = \frac{pF'(N) - w}{F'(N)} = -\frac{D(p)}{D'(p)}, \text{ so that } \frac{pF'(N) - w}{F'(N)} = \frac{p}{\eta},$$

where  $\eta = -pD'(p)/D(p)$ . It follows that p and N are determined by the two equations

(7) 
$$p = \mu w / F'(N),$$

(8) 
$$F(N) = D(p),$$

where  $\mu = 1/(1-1/\eta)$ . Thus, the fundamental problem with (7) is not that it is false but that it only indirectly affects employment, through its effect on the product price, while the direct determinants of employment are incorporated in (8).

Let us next introduce a useful first approximation. Suppose that the marginal productivity is constant, F'(N) = a, up to a certain employment level equal to  $\hat{N}$ , where it begins to fall. If the fall is very strong, then output cannot be much higher than  $a\hat{N} = k$ , which consequently characterizes the firm's *capacity*. This example is not only useful as a bench-mark but also rather realistic, as argued, for example, by Layard et al. (1991 p. 340).<sup>3</sup>

Of course, a firm's supply curve is not vertical for high prices even if it usually is rather steep due to constraints on employment in existing premises and with existing machinery (and restrictions on overtime etc.). Capacity is consequently in general not a parameter but an increasing function of the price. However, assuming a constant capacity simplifies the analysis considerably without changing its substance.

To initiate sales the firm has to announce a price in a market where buyers take prices as given, as in most consumer markets. If the firm anticipates that product demand will be low in relation to its capacity, it will announce  $p = \mu w/a$ . This formula shows how the firm adjusts its price to changes in wages and productivity for a given mark-up, while the mark-up is adjusted by the firm according to its perceptions of the price elasticity of product demand. And the firm adjusts its employment according to (6) as long as  $D(\mu w/a) \le k$ . Thus,

(9) 
$$p = \mu w/a \text{ and } N = D(p)/a \text{ if } D(p) \le k$$
.

If on the other hand  $D(\mu w/a) > k$ , price and employment are adjusted by the firm, assuming perfect information on the demand function, until the equations (7) and (8) are satisfied. When the fall in marginal productivity is very strong for employment above  $\hat{N}$ , employment is (approximately) equal to  $\hat{N}$ , while the price is raised until (quantity) rationing has been eliminated. Thus, as a first approximation,

(10) 
$$p = D^{-1}(k) \text{ and } N = k/a \text{ if } D(\mu w/a) > k$$

According to (10), variation in product demand will only affect prices but not employment in a boom, if capacity constraints are binding. This is, in general, only a first approximation. But it does represent the reasonable notion that a firm's employment can only increase marginally when its capacity has been reached. Formally, this marginal adjustment involves (7), but (7) only applies to a very small interval of employment, assuming that F'(N)

<sup>&</sup>lt;sup>3</sup> See also, for example, Blinder et al. (1998 p. 102) on the prevalence of constant marginal cost.

declines rapidly above  $\hat{N}$ . And during this adjustment of employment the price will also rise, according to (7) and (8).

We conclude that, as a first approximation, employment is *never* determined by (2), at least not directly. Employment is determined *indirectly* by (2) in a recession, since then (2) determines the product price, while employment is determined by the demand for the firm's output at this price and the firm's labour productivity, according to (9). And in a boom (2) has a minor effect on employment, provided that production and employment are restricted by the firm's capacity, according to (10). While (7) and (8) constitute a complete model of price and employment at the firm level, (9) and (10) constitute a useful first approximation, when a firm's production function can be characterized by two parameters, its labour productivity (a) and its capacity (k),

#### 3. Labour demand of a competitive firm

These results for a noncompetitive firm generalize easily to a competitive firm, but only if we complete the model by adding the fact that a competitive firm must be part of a competitive industry. For then we can see that not only (2) but also (1) is incomplete. In fact, price p and employment N of a firm in a competitive industry with a finite number (n) of identical firms with production function F and a wage level equal to w are determined by the equations

$$(11) p = w/F'(N)$$

(12) 
$$nF(N) = D(p),$$

where D is the industry's product-demand function and  $F'' \leq 0$ .

In this complete model of a representative competitive firm, eq. (11) models the notion of a price-taking firm which is in equilibrium only when price equals marginal cost. And eq. (12) models not only the notion that supply equals demand in equilibrium but also the fact that the number of firms is always finite in an industry, even if the industry is competitive. Eq. (12) also implies that every firm has the same market share, but this assumption is made for simplicity only.

Now, if the marginal productivity is constant, F'(N) = a, (11) completely determines price, while (12) determines employment. And this happens if product demand is low, so that (13) p = w/a and N = D(p)/na if  $D(p) \le nk$ ,

where k denotes a firm's capacity, as in Section 2. In a recession production and employment can consequently be restricted by sales even in a competitive industry. And then an individual

firm will also be restricted by sales, or more precisely by its market share, which in our simple example with identical firms is 1/n. (If firms are identical, and no capacity constraint is binding, the probability that a consumer chooses to buy from a particular firm is 1/n if there are *n* firms, and it follows from the law of large numbers that each firm's market share is 1/n.) Note that this possibility is excluded *by assumption* in the traditional model of employment in a competitive firm in the short run.

Moreover, capacity constraints (F'' < 0) will *raise* prices (since p = w/F'(N) > w/a) but *reduce* the effect of wage changes on employment. In fact, as a first approximation,

(14) 
$$p = D^{-1}(nk)$$
 and  $N = k/a$  if  $D(w/a) > nk$ 

In a boom production and employment will consequently be restricted by capacity (if the boom is sufficiently strong) and the market price will be the market-clearing price. And employment will be constant or only marginally affected by (11).

Since (11) or, equivalently, (1) is so firmly established in the literature, it is perhaps hard to accept that it determines employment in a competitive firm only partly and indirectly, through its determination of the market price in a recession. The marginal-productivity function is of course a basic determinant of employment in a firm. But it determines employment essentially through two parameters, namely its labour productivity (*a*) and its capacity (*k*). At least this is true as a first approximation. I have also argued that this first approximation is probably rather good, and it is certainly helpful for the intuition. But note that it is not crucial for my argument. Eq. (1) is an incomplete model of employment in a competitive firm even if F''(N) < 0 for every N.

Note also that, apart from the mark-up, results are the same for a competitive and a noncompetitive firm. This is because I have relaxed an implicit assumption of the traditional model of a competitive market, namely that a price-taking firm can never be restricted by what it can sell. The necessity to relax this assumption is most obvious with constant returns, when production must be restricted by sales in the firm's industry and hence also in the industry's firms.

Now, even if the basic principles of labour demand can be presented with labour as the only input, it remains to make the model completely complete by adding intermediate goods and recruitment activities.

#### 4. Intermediate goods

Suppose that not only employment is proportional to output q, N = q/a, but also other variable inputs, M = q/b, so that variable costs can be written as

(15) 
$$C = (w+g)N, \text{ where } g = va/b.$$

Thus we assume that even variable costs other than labour costs are proportional to employment, with an addition to the wage rate (g) which depends on the prices of additional inputs (v) as well as the relation between employment and other inputs (b/a = N/M). The parameter g reflects not only the choice of technology as characterized by the relation between employment and other inputs (b/a) but also the prices of other inputs (v).

We also assume, to begin with, that not only a firm's capacity but also its technology is fixed in the short run. Even if a firm in the long run attempts to reduce its marginal cost c = (w+g)/a at anticipated input prices by an appropriate substitution between labour and other variable inputs (like semi-finished goods), it can hardly change this mix instantaneously when input prices are revised. A firm can adjust its output prices almost instantaneously to new input prices, but it takes longer to change its technology.

With these assumptions it follows that

(16) 
$$p = (1+m)(w+g)/a \text{ and } N = D(p)/a \text{ if } D(p) < k$$
,

where *p* denotes the price the firm sets, D(p) the firm's sales at this price, and *m* is a mark-up which equals 0 for a firm in a competitive market and  $1/(\eta - 1)$  for a monopoly.

Thus, a firm's production is restricted either by its sales, according to (16), or by its capacity. And if production is restricted by capacity, then employment is also restricted by capacity, implying that the wage level and other elements of marginal cost have a negligible impact on employment (at least as a first approximation) unless (16) applies.

Note that the cost of intermediate goods will reduce the wage elasticity of labour demand, in accordance with Marshall's laws of derived demand (Marshall 1982 p. 319). In fact, as differentiation of (16) shows (see Appendix), the wage elasticity of labour demand is equal to the price elasticity of product demand multiplied by labour's share in total variable costs, w/(w+g), provided, of course, that production is restricted by sales and not capacity.

The production technology is characterized by three parameters, namely capacity (k), labour productivity (a = q/N) and input technology (a/b = M/N). Labour productivity is a summary measure of the real effect of labour, machinery and intermediate goods in the production of goods, while input technology (the relation between intermediate goods and employment) measures the firm's dependence on current production in other firms.

Note finally that I have shown how product demand affects labour demand *in the short run*, at given capacities, technologies and skills. If, however, technology, organization or real capital can be changed as a response to a change in wages, we have to distinguish between 'substitution effects' and 'scale effects'.<sup>4</sup> For example, a rise of the wage level may get the firm to substitute intermediate goods or services for some employee-hours in the intermediate run or new machines in the long run. Or a rise of wages for some workers may get the firm to substitute one type of workers for another type. In any case, if a change of the wage level or the wage structure leads to substitutions in some respects, this may have a substantial indirect effect on employment after some time.

In general, and in the long run, substitutions may affect not only intermediate goods (g) and labour productivity (a) but also the average wage (w), implying that the effect on marginal cost (c = (w+g)/a) in the long run may differ from the short-run effect of a change in w. In the short run we may have a scale effect from a change in wage level on the product price and hence on production and employment, while substitution effects of various kinds may modify both price and labour productivity and hence also employment in the long run. Note, however, that at every point in time price and employment are determined by (16) with current values of the determinants.

### 5. Recruitment costs

If a firm expects an increase in the demand for its products to be only temporary, it will not necessarily increase its employment, due to costs of hiring and firing, as elaborated in the literature on adjustment costs.<sup>5</sup> But recruitment costs can reduce employment even in a steady state (where we can ignore firing costs), provided they by raising product prices also reduce demand for the firm's products.

To see how recruitment costs add to variable costs we begin by observing that the variable costs for a firm with employment N in general can be written as

(17) 
$$C = wN + gN + \alpha H + \gamma V,$$

where H denotes the firm's number of hires per period and V its stock of job vacancies. The wage level is denoted by w, other variable production costs (including costs of raw materials

<sup>&</sup>lt;sup>4</sup> On the distinction between substitution effects and scale effects related to the choice of labour and capital in the long run, see, for instance, Section 1.2 in Chapter 4 in Cahuc and Zylberberg (2004).

<sup>&</sup>lt;sup>5</sup> See, in particular, Nickell (1986) and Hamermesh (1993).

and other inputs in the production process) are measured by gN, as in Section 4, while recruitment costs are captured by the parameters  $\alpha$ , as in Nickell (1986), and  $\gamma$ , as in Pissarides (1990).

Note that recruitment costs are in general composed of both *hiring costs* ( $\alpha$  per hire) and *vacancy costs* ( $\gamma$  per job vacancy and period). Hiring costs include costs of introduction and training but also costs of job advertising if these are concentrated to the beginning of the recruitment process, while vacancy costs include recruitment costs which increase with the length of recruitment, like a fee to a private employment agency if the firm is paying the agency for its services per week and not per job match.

Moreover, the stock of job vacancies (*V*) is related to the flow of hires (*H*) according to (18) V = bHT,

where b denotes the share of hires preceded by job vacancies, so that bH measures the inflow of job vacancies, and T denotes the average duration of job vacancies. The parameter b is included because much hiring is not mediated through job vacancies as measured in vacancy surveys, as shown by Davis, Faberman, and Haltiwanger (2013) for the U.S. This includes in particular instantaneous hires, like recalls of former employees.

We assume in this paper – as in the literature on adjustment costs – that *all* hires are instantaneous, including hires preceded by job vacancies. To see why, note first that firms create job vacancies (recruitment processes) in order to avoid unfilled jobs (unmet labour demand). Thus, firms start recruiting in anticipation of future needs. And if, for instance, a separation can be anticipated and a replacement made before the separation, then replacement is instantaneous even if recruitment is not.

Note that eq. (17) already excludes unfilled jobs, because employment is assumed to be constant over time in (17). In other words, this paper deals with the effect on employment of costly search on the simplifying assumption that firms completely control employment. This may be a reasonable approach if unfilled jobs are rare and hard to predict, so that firms simply ignore them when prices are adjusted to recruitment costs.<sup>6</sup>

The approach may also be reasonable for employers who anticipate problems to keep employment constant, provided it incorporates plans to use substitutes (including personnel from temporary work agencies) whenever substitutes are necessary during recruitment of

<sup>&</sup>lt;sup>6</sup> But even if we can ignore unfilled jobs when estimating the indirect effect of recruitment costs on employment, we cannot ignore the fact that unfilled jobs reduce employment directly by creating a gap between labour demand (desired employment) and employment. The measurement of unfilled jobs as distinct from job vacancies is consequently an important problem, but it is not addressed in this paper, which focuses on labour demand, not *unmet* labour demand.

replacements in order to avoid unfilled jobs. But then anticipated costs of the necessary substitutes must be added to vacancy costs, even if only vacancy costs *above* the wage level w can be included, since wN in (17) already includes the costs of having posts occupied.

Now, in a steady state, with product demand and other market conditions assumed to be constant for some time, a firm only has to replace separations. Separations occur for a variety of personal or institutional reasons and are not necessarily proportional to a firm's number of employees. However, for a representative firm we may assume that

$$(19) H = sN,$$

where *s* denotes the average separation rate for the group of firms considered.

It follows from (19) and (18) that

(20) 
$$\alpha H + \gamma V = \alpha s N + \gamma b H T = \alpha s N + \gamma b s N T$$

and substituting this into (17) we find that

(21) 
$$C = (w + g + s(\alpha + \gamma bT))N$$

and hence that

(22) 
$$p = (1+m)(w+g+s(\alpha+\gamma bT))/a \text{ and } N = D(p)/a \text{ if } D(p) < k$$

where *p* denotes the price the firm sets (and *m* is a non-negative mark-up) and D(p) denotes its sales at this price, while *a* denotes the firm's labour productivity and *k* its capacity. And if sales are restricted by capacity, then production and employment are also restricted by capacity, implying that recruitment cost have no impact on employment unless (22) applies – which consequently is the only case we have to consider. Note that with a separation rate equal to 36 per cent of employment per year we have s = 0.03 per month, so for recruitment costs to have a noticeable impact on the product price according to (22), they have to be rather large compared to the monthly wage (*w*) and other direct costs (as measured by *g*).

### 6. Aggregation

To sum up, unless a firm's production and employment are restricted by its capacity (k), price (p) and employment (N) in a firm are determined by

(23) 
$$p = (1+m)(1+h)(w/a)$$
 and  $N = D(p)/a$  if  $D(p) < k$ ,

where w is the wage level, a labour productivity, h variable costs other than labour costs as a share of labour costs, and D(p) the firm's sales at the price it sets.

Aggregation from a single firm to its industry is straightforward. Employment in the industry is determined as the sum of employment in its firms at a common market price (the

law of one price). (In a market with differentiated goods prices may differ somewhat but the price level must be the same.) This means that mark-ups are not necessarily set independently by every firm. For example, in a market with price leadership all firms but one take the market price as given, and then the mark-up for a price taker is determined by the market price p set by the price leader (maximizing its individual profits) and the marginal cost c of the price taker, m = (p-c)/c.

If production is restricted by capacity in every firm, then the industry's employment is also restricted by capacity and in fact determined by capacity and labour productivity. And if production is restricted by sales in every firm, then the labour demand curve of the industry – relating the industry's employment to its wage level – has the same elasticity as the labour demand curve of a constituent firm if all firms are identical.

When firms are different the situation is more complicated, not only because the parameters in (23) may differ between firms, but also because production can be restricted by capacity in some firms and by sales in other firms. But in all cases there should be a negative relation between wages and employment, provided that production is restricted by sales for at least some firms, since then prices will depend on wages, implying that sales, production and employment will also depend on wages.

At every stage of an industry's evolution its production and employment are restricted either by sales or capacity. Focusing on wages, employment is sensitive to wages if they are sufficiently high, so that production and employment are restricted by sales and not capacity. And then higher wages will by raising prices reduce sales, production and employment and the effect depends on the price elasticity of the demand for the industry's products but also on labour's share in total variable costs.

Note finally that the basic reason for a negative relation between wages and employment at the microeconomic level is that at this level we can assume that the demand for an industry's products is independent of the industry's wages. And this assumption is not necessarily valid at the macroeconomic level.

#### 7. Conclusions

There is, of course, a negative relation between nominal wages and worker-hours for a firm and its industry. However, as shown in this paper, this is not because of a declining marginal product of labour, but because higher wages raise product prices and reduce sales, production, and employment. In fact, the main effect of a declining marginal product of labour is to raise product prices during a boom when production is restricted by capacity and not sales.

Moreover, unless a firm's production and employment are restricted by its capacity, price and employment in a firm are determined by (23). Thus, a firm's labour demand depends on product demand, capacity, labour productivity, wages and other direct costs, but also on the mark-up on direct costs chosen by the firm.

In practice there are, of course, some complications. Production does not adjust perfectly to sales unless sales precede production (production to orders). In markets where production precedes sale, as in most consumer markets, production will in general differ from sales, even if changes in inventories are negligible. And adjustment costs will stabilize employment when sales are variable or hard to predict. But this does not change the basic message in this paper, namely that a firm's employment depends on the demand for its products and its nominal wages, in addition to its capacity, labour productivity and variable costs other than wages. A firm's decisions on employment are not based on real wages.

# Appendix. Proof of Marshall's third law of derived demand.

Suppose that N = D(p)/a where p = (1+m)(w+g)/a, as in Section 4. The wage elasticity of labour demand is defined as

(A1) 
$$\eta_L = -\frac{dN/N}{dw/w} ,$$

while the price elasticity of product demand is defined as

(A2) 
$$\eta = -\frac{dD/D}{dp/p}.$$

To prove that

(A3) 
$$\eta_L = \frac{w}{w+g} \eta,$$

we first note that

(A4) 
$$dN/N = D'(p)dp/D(p) \text{ and } dp = (1+m)dw/a = \frac{p}{w+g}dw,$$

so that

(A5) 
$$\frac{dN/N}{dw/w} = \frac{\left(D'(p)/D(p)\right)dp}{dw/w} = \frac{\left(D'(p)/D(p)\right)pdw}{\left(dw/w\right)\left(w+g\right)}.$$

It follows from (A1 and (A5) that

(A6) 
$$\eta_L = -\frac{w}{w+g} p D'(p) / D(p) = -\frac{w}{w+g} \frac{p(dD/dp)}{D} = -\frac{w}{w+g} \frac{dD/D}{dp/p}$$

and (A3) follows from (A6) and (A2).

### References

Borjas, George J. 2008. Labor Economics, Fourth Edition, McGrawHill.

Blinder, Alan S., Elie R. Canetti, David E. Lebow, and Jeremy B. Rudd 1998. *Asking About Prices. A New Approach to Understanding Price Stickiness*, Russel Sage Foundation.

Cahuc, Pierre and André Zylberberg 2004. Labor Economics, MIT Press.

Davis, Steven J., R. Jason Faberman, and John C. Haltiwanger 2013. "The Establishment-Level Behavior of Vacancies and Hiring." *Quarterly Journal of Economics* 128: 581-622.

Hamermesh, Daniel S. 1993. Labor Demand, Princeton University Press.

Layard, Richard, Stephen Nickell and Richard Jackman 1991. Unemployment. Macroeconomic Performance and the Labour Market, Oxford University Press.

Lindbeck, Assar 1998. "New Keynesianism and Aggregate Economic Activity", *Economic Journal*, 108, 167-180.

Marshall, Alfred 1982. *Principles of Economics*, Eighth Edition, Macmillan (first printed in 1920).

Nickell, Stephen J. 1986. "Dynamic Models of Labour Demand". In Ashenfelter, Orley, and Richard Layard (eds.). *Handbook of Labor Economics*. North-Holland.

Pissarides, Christopher A. 1990. Equilibrium Unemployment Theory, Basil Blackwell.