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A THEORY OF VACANCIES

by

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Abstract: This paper is, firstly, a reappraisal of the matching function, arguing that the proper specification of the relation between hirings (*H*), vacancies (*V*), and unemployment is the *duration function*, which shows how average recruitment times as measured by *V/H* depend on unemployment and other relevant variables. Secondly, indirect effects of longer recruitment times on employment through higher recruitment costs are studied by extending previous models to include both price formation and the distinction between vacancy costs and hiring costs. Thirdly, direct effects of longer recruitment times on employment through more unfilled jobs are explored and illustrated with data from Sweden since 2000 from a new business survey, which measures not only job vacancies but also unfilled jobs.

Keywords: Vacancies, hirings, matching function, friction

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> "In a world where economists have little certain knowledge, the shift of the *U/V* curve provides us with vital clues to the sources of the rise in unemployment. Large shifts indicate that a major part of the rise is due to changed behaviour of workers and employers in the filling of vacancies." (Layard, Nickel, and Jackman 1991 p. 220)

> "Frictions have made important inroads in modern macroeconomics. In the labour market they are used to explain the existence of unemployment … . In the majority of cases, the modeling tool used to capture the influence of frictions on equilibrium outcomes is the aggregate matching function. … The matching function … occupies the same place in the macroeconomist's tool kit as other aggregate functions, such as the production function and the demand for money function." (Petrongolo and Pissarides 2001 pp. 390-91)

> "The search approach has led to a focus on dynamics in understanding the labor market. We now know quite a bit about these flows and their variation with regressors and across demographic groups. 'Structural' interpretation of these flows is more problematic. In the Markovian framework we are accustomed to, it is these flow rates that determine the equilibrium or natural unemployment rate. Policies affecting the flow rates are likely to have long-term effects; those focused on the stock at a particular period in time will have only short-term effects. This notion has been crucial in the discussion of unemployment insurance policy and its effect on the natural rate of unemployment. Of course, the intuition leading to this generalization is based on Markovian (or semi-Markovian) models, and this specification is rarely checked." (Devine and Kiefer 1991 p. 307)

1. Introduction

The matching function is a relation between hirings, vacancies, and unemployment, which reflects friction in job matching. The relation has been interpreted and estimated as a production function, with stocks of vacancies and unemployment as inputs and the number of hirings per period as output, first by Pissarides (1986), Blanchard and Diamond (1989), and Layard, Nickel and Jackman (1991), and then by many others, as the comprehensive survey by Petrongolo and Pissarides (2001) shows.

A shift of the matching function with fewer hirings at given inputs of vacancies and unemployment suggests a change of the matching technology which reduces the number of hirings per period, and which consequently also reduces the arrival rate of job offers which face the unemployed and increases unemployment by prolonging unemployment spells.

However, this flow approach to the effect of friction on unemployment is misleading. Friction in job matching does not affect unemployment directly, by prolonging unemployment spells, but indirectly, by reducing employment, as I shall argue in this paper.

According to the dynamic theory of labour demand, the number of hirings per period chosen by a firm when hirings are *instantaneous* is in general determined by the firm's desired net change of employment and its need to replace workers who, for various reasons, are leaving the firm. And the firm's desired net change of employment depends on 1) current employment, 2) current sales, prices and costs (including recruitment costs), and 3) expectations of future sales, prices and costs, as elaborated, for instance, in Nickell (1986) or Hamermesh (1993). Thus, individual hirings, and hence also aggregate hirings, are functions of current and expected prices, sales and costs, including recruitment costs.

This is the established microfoundations of the dynamic theory of labour demand if hirings are instantaneous. How will the introduction of vacancies modify these foundations?

If all hirings were instantaneous there would be no vacancies as measured in vacancy surveys. We can consequently interpret the number of vacancies as a summary measure of friction in terms of deviations from instantaneous hirings. In fact, the vacancy rate was suggested as a measure of friction already in the seminal paper by Dow and Dicks-Mireaux (1958 p. 3), where a declining vacancy rate was interpreted as a fall in labour maladjustment.

Of course, controlling for hirings is preferable whenever information on hirings is available, since a high job vacancy rate may also be due to a rise in job turnover or job reallocation, as emphasized by, for instance, Thomson (1966 p. 191), Abraham (1987), and Blanchard and Diamond (1989). In fact, dividing the number of job vacancies (V) by the number of hirings per period (H), we obtain a measure of the average duration of recruitment.

In this paper I propose an analysis of the effect of friction on employment and hirings in three steps.

First, factors which create deviations from instantaneous hirings as measured by the average duration of recruitment are identified by estimating the *duration function*, that is, by regressing V/H on unemployment and other potentially important variables.

Second, indirect effects of longer recruitment times on employment and hirings through higher recruitment costs are analysed by extending previous models in Nickel (1986) and Pissarides (2000) to include price formation and the distinction between vacancy costs and hiring costs. We find that longer recruitment times reduce employment if they raise recruitment costs so much that sales and employment are reduced by higher prices. We also find that longer recruitment times will raise the number of vacancies. Hirings are affected only if employment is affected.

Third, friction in job matching can reduce employment even if recruitment costs are negligible, namely by making the number of filled jobs (employees) less than the number of jobs, as elaborated in Section 5. This direct effect of friction on employment can only be identified by extending traditional vacancy surveys to include not only job vacancies but also unfilled jobs.

The paper is organized as follows. Job vacancies are defined and related to hirings in Section 2, observing that there are hirings without vacancies (probably many) as well as vacancies without hirings (probably few). In Section 3 measures of friction, including not only the rate of job vacancies and the average duration of job vacancies but also the rate of unfilled jobs, are illustrated with data from Sweden since 2000 from a new business survey. Indirect effects of longer recruitment times on employment through higher recruitment costs are analyzed in Section 4, while direct effects of longer recruitment times on employment through more unfilled jobs are discussed in Section 5. Section 6 concludes the paper.

2. Vacancies and hirings

Some hirings are made more or less directly, for example, by recalling workers previously laid off or by offering jobs to spontaneous job applicants. In other cases there is no existing pool of job applicants which a firm can turn to. Instead the firm has to attract new job applicants by advertising its demand for personnel in newspapers or other media, by placing job orders with a public or private employment agency, or by contacting potential candidates directly. And then vacancies understood as ‘recruitment processes’ arise, as discussed in, e.g., van Ours and Ridder (1992) and Burdett and Cunningham (1998).

More precisely, a *job vacancy* begins when a firm starts to recruit a worker and it ends when a worker offered the job accepts it (or when recruiting is discontinued for other reasons). This is also the usual definition in vacancy surveys, including all the surveys discussed in NBER (1966), Muysken (1994), and Verhage et al. (1997).

2.1. Hirings without vacancies

Can we assume that every hiring begins with a job vacancy? This assumption is implicitly made when attempts are made to estimate the total number of job vacancies (V) from the total number of hirings per period (H) and the average duration of job vacancies (T) for some part of the economy according to the formula

$$V = HT, \tag{1}$$

as in, for instance, Abraham (1983) and Jackman, Layard, and Pissarides (1989). In this context the assumption is true by definition. But in general it may be difficult to define and measure job vacancies for some types of hirings.

Suppose, for example, that a firm is so big that it has to hire 10 workers per month to replace a constant flow of 10 separations per month, and that each month the firm hires those ten applicants who contact the firm first (assuming they are properly qualified). Then it may be difficult for the firm to say when it started to recruit a particular worker, since it always has some recruitment activities going on. In other words, the firm may find it difficult to specify the number of 'job vacancies' it has on a particular day, but easy to specify the number of (desired) hirings per month.

Moreover, some hirings may occur without any preceding recruitment activities at all, for example, when a job applicant contacts an employer who then decides to hire. In this case one might say that 'supply creates its own demand' when a properly qualified person turns up. And then there is no job vacancy (recruitment process) which precedes hiring.

Note also that even if a recruitment process precedes a hiring, it can sometimes be so short that the distinction between the recruitment process and the hiring is negligible. Examples include recalls by phone calls of former employees, or selection of casual labourers in a hiring hall at the beginning of a day. And in surveys on job vacancies firms may not consider recalls to be recruitment, even if the time between job offer and acceptance is long, unless instructed to do so.

Hirings with negligible or non-existent recruitment processes before hiring may be called *instantaneous hirings*. Information on such hirings is scarce. But what information there is does suggest that not every hiring begins with a job vacancy. Consider, for instance, the Employment Opportunity Pilot Project (EOPP) surveys in the United States in 1980 and 1982.[1] In these surveys employers were asked questions about the hiring process for the most recent newly hired person. And in the first survey 28 percent responded that they did not recruit for the position.

In general we consequently have

$$V = \beta HT, \tag{2}$$

where β denotes the proportion of hirings preceded by job vacancies. Note that this formula can be used to estimate the proportion of instantaneous hirings ($1-\beta$) in a labour market, provided, of course, that we have information on not only H and T but also V.

2.2. Vacancies without hirings

Equation (2) presupposes not only a steady state but also that all vacancies end in hiring, so that βH equals the inflow of vacancies. But job vacancies can be cancelled by firms before hiring. This can happen simply because the situation has changed, so that the firms no longer want to recruit new personnel, or because firms having difficulties in forecasting their labour demand realize that they have exaggerated their needs.[2] Or some job vacancies may be cancelled because firms realize that no recruitment is possible at the moment and that they have to solve their staffing problems by other means, for instance, reorganization and training followed by posting job vacancies which are easier to fill.

Information on cancelled job vacancies is scarce, particularly on firms' *reasons* for cancelling job vacancies. But some sample surveys by the Public Employment Service in Sweden in the beginning of the 1990's suggest that the proportion of job vacancies which end in hiring is very high, and *at least* equal to 90 percent.[3] Results by van Ours and Ridder (1992 p. 145) suggest the same thing.

Thus, on one hand, it seems to be approximately true that all job vacancies (sooner or later) end in hiring, which is a standard assumption in the search literature,

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[1] Results from the first wave are reported in Barron, Bishop and Dunkelberg (1985) and results from the second wave in Barron and Bishop (1985).
[2] As emphasized by, for instance, Thomson (1966 p. 177).
[3] See Falk (1996).

including Pissarides (2000). On the other hand, more information is needed, either to corroborate this assumption, which is in stark contrast to the fact that usually almost half of unemployment spells end without hiring, or to throw light upon another important aspect of friction.

2.3. The relation between hirings and vacancies

Equation (2) shows how the stock of vacancies (*V*) adjusts to variations in the inflow of vacancies (βH) and their average duration (*T*). And in general both β and *T* depend on unemployment (as well as other variables characterizing labour supply, search efficiency, and institutional factors).

Of course, if (2) is a stable empirical relation between vacancies, hirings, and unemployment, then

(3) $$H = V/\beta T = m(V,U)$$

will also appear as a stable empirical relation between hirings, vacancies, and unemployment. In the search and matching literature this relation has been interpreted as an *aggregate matching function*, showing how vacancies and unemployment as 'inputs' give rise to 'output' in the form of hirings.

Equation (3) is a specification which suggests that vacancies – as well as unemployment and other variables on the right hand side of (3) – may have *direct* effects on hirings. But hirings are not determined directly by vacancies and unemployment. Hirings are determined by firms' need to replace separations and realize changes in employment, conditional on (current and expected) prices, sales and costs, including recruitment costs. This is elaborated in Nickel (1986) for *instantaneous hirings*. It remains to see how *deviations from instantaneous hirings* affect hirings and employment. And the first step in this investigation is to develop *measures of deviations from instantaneous hirings*.

3. Defining and measuring friction

In this section I survey three measures of friction, namely the rate of job vacancies, the average duration of recruitment, and the rate of unfilled jobs, and I illustrate these measures with data from Sweden since 2000 from a new business survey.

If all hirings were instantaneous there would be no vacancies as measured in vacancy surveys. We can consequently interpret the vacancy rate as a summary measure of friction in terms of deviations from instantaneous hirings. And the vacancy rate is closely related to the *U/V* curve, which is the classical point of departure for the analysis of friction in the labour market

3.1. Job vacancies

Movements of the *UV* point in a *UV* diagram are easy to explain starting from equation (2), which says that the stock of vacancies is proportional to the flow of hirings.

As aggregate demand increases, hirings and vacancies will increase as employment (*N*) increases and unemployment falls, which implies a negative relationship between unemployment and vacancies. More precisely, since $H = S + \dot{N}$, where *S* denotes separations and $\dot{N}$ the growth rate of employment, we have

$$V = \beta T \left(S + \dot{N} \right), \quad (4)$$

where $\dot{N}$ is positive during an upswing, when unemployment decreases, and negative during a downswing. Thus, during a business cycle the *UV* point moves counter-clockwise around an equilibrium locus defined by the equation

$$V = \beta TS, \quad (5)$$

where, in general, not only βT but also *S* depends on unemployment.

It is useful to distinguish between the *observed UV* curve, which might be called the *UV loop*, defined by (4), and the *equilibrium UV* curve, often called the *Beveridge curve*, defined by (5). The negative relationship between vacancies and unemployment during a business cycle which is actually observed in a *UV* diagram is always a part of the *UV* loop.

While shifts of the *UV* loop between business cycles may be due to shifts of the level of aggregate demand, shifts of the Beveridge curve reflect structural changes. But an outward shift of the Beveridge curve, that is, more vacancies at a given unemployment rate, is not necessarily due to longer vacancy spells or fewer instantaneous hirings. It may also be due to more separations.

Regressions of the vacancy rate on unemployment, employment growth, and other relevant variables may show, for instance, that variation in unemployment benefits has a well-defined effect on friction as measured by the vacancy rate. But there are better measures.

3.2. Average recruitment times

Dividing the number of vacancies (*V*) by the number of hirings per period (*H*), we obtain a measure of the average duration of recruitment.

Table 1 *Job vacancies, unfilled jobs, and average recruitment times in Sweden. Quarterly averages of monthly figures in the private sector.*

Period	Employment (thousands)	Hirings (thousands per month)	Job vacancies (thousands)	Job vacancie s (per cent)	Average duration (months)	Unfilled jobs (per cent)
2000 Q3	2550	88.4	69.2	2.7	0.81	1.1
Q4	2493	72.2	63.1	2.6	1.05	1.1
2001 Q1	2504	70.3	57.9	2.3	0.93	1.0
Q2	2538	91.8	49.4	2.0	0.58	0.8
Q3	2647	83.2	44.7	1.7	0.55	0.7
Q4	2574	65.1	38.8	1.5	0.65	0.6
2002 Q1	2522	61.4	38.4	1.6	0.64	0.6
Q2	2573	97.7	39.1	1.6	0.40	0.6
Q3	2575	84.1	35.2	1.4	0.43	0.6
Q4	2556	72.1	31.9	1.3	0.49	0.5
2003 Q1	2498	63.6	36.0	1.5	0.61	0.7
Q2	2562	99,7	30.6	1.2	0.34	0.4
Q3	2563	75.0	26.4	1.0	0.36	0.4
Q4	2518	60.3	23.7	0.9	0.39	0.4
2004 Q1	2458	57.9	25.7	1.0	0.44	0.4
Q2	2547	96.2	27.8	1.1	0.29	0.4
Q3	2563	73.8	26.8	1.1	0.37	0.4

Notes: Columns 5 and 7 report job vacancies and unfilled jobs as per cent of employment. Column 6 reports ratio estimates of *V/H* (not estimates in column 4 divided by estimates in column 3). Standard deviations are approximately 0.05 in columns 5 and 7 and at most 0.03 in column 6.

Source: Statistics Sweden (business surveys on employment and vacancies). See SCB (2004a) and SCB (2004b) for examples of reports from the quarterly surveys.

Table 1 reports vacancies and average recruitment times in Sweden since 2000 according to a new business survey. It confirms the stylized fact that the average completed duration of a vacancy is in most cases under a month.[4]

Note, however, that *V/H* is a measure of the average duration of all vacancies, including ‘vacancies with negligible duration’, like recalls by phone calls of former employees. Since such vacancies are not usually measured in vacancy surveys, it is more accurate to say that *V/H* equals the average duration of vacancies (*T*) multiplied by the proportion of non-instantaneous hirings (β), since $V/H = \beta T$ according to (2). If, for instance, a separate sample survey on vacancy durations shows that the average spell of a (completed) vacancy is 3 months, then the proportion of instantaneous hirings ($1-\beta$) is 2/3 if *V/H* is equal to 1 month.

In general the average duration of recruitment as measured by *V/H* depends on labour supply, search efficiency, and institutional factors. This dependence can be characterized by a *duration function*, where the independent variables include not only unemployment but also other relevant variables.

Estimation of the duration function is an important first step in the analysis of the effect of friction on employment. But it remains to find out how recruitment times affect employment.

3.3. Unfilled jobs

Note first that the fact that it takes time to recruit workers does not *necessarily* reduce employment. For, as emphasized by Holt and David (1966 p. 82), firms create vacancies in anticipation of future needs. If, for instance, a separation can be anticipated and a replacement made before the separation, then *replacement* is instantaneous even if *recruitment* is not. Otherwise an *unfilled job* exists from the day the employer wants the worker to start to the day the worker starts.

The number of unfilled jobs is a measure of the direct effect of friction on employment. More precisely, as elaborated in section 5, the time it takes to recruit workers reduces employment by making the number of filled jobs (employees) less than the number of jobs. And the difference is the number of unfilled jobs.

It is sometimes assumed (for simplicity in theoretical models) that every separation gives rise to an unfilled job, or that firms have to create unfilled jobs in order to recruit workers.[5] But, as noted above, many separations are anticipated and replacements made before the corresponding jobs become unfilled. This applies particularly to large firms.[6]

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[4] See, for instance, Petrongolo and Pissarides (2001 p. 421).
[5] See, for instance, Pissarides (2000 p. 5), where it is assumed that ‘only vacant jobs can engage in trade’.
[6] As emphasized, for instance, by van Ours and Ridder (1992 p. 140).

In fact, according to the Swedish vacancy survey, which measures not only job vacancies but also unfilled jobs, only about 40 per cent of the job vacancies are unfilled jobs, as shown by Table 1.

Unfilled jobs are defined more precisely as *unoccupied job vacancies which are available immediately* in the Swedish vacancy survey. This is a definition which excludes occupied job vacancies and job vacancies to be filled later, in the same way as the definition of unemployed workers in labour force surveys excludes job seekers with a job and job seekers without a job who cannot start work until later.[7]

4. Indirect effects of recruitment costs on hirings and employment

A formal analysis of the effect on hirings of vacancies is in general complex, as is easily seen by adding the number of vacancies as a control variable to the already complex analysis in Nickel (1986). But it is reasonable to assume that the order of magnitude of the effects of vacancies on hirings is captured by an equilibrium analysis. And then we can ignore hirings which realize the firm's desired net change of employment – including the complex dependence on expectations – and focus on those hirings which replace separations.

In general separations – and hence also hirings – depend not only on employment but also on the state of the labour market, as measured, for instance, by the unemployment rate, and on structural factors, including, in particular, the share of temporary jobs, as emphasized by, for instance, Verhage et al. (1987 p. 17). But we shall here focus on the dependence on employment. Moreover, vacancies can influence a firm's separations either by affecting its number of employees or its separation rate. Assuming, as a first approximation, that the effect of vacancies on the separation rate is negligible compared to their effect on the number of employees, we shall here focus on effects of vacancies on the number of employees and therefore postulate a constant separation rate.

4.1. Assumptions

In equilibrium with recruitment costs a firm's flow of profits is

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[7] The definition and measurement of unfilled jobs in the Swedish vacancy survey is explained in more detail in Appendix 1.

(6) $$\pi = R(N) - wN - \alpha H - \gamma V ,$$

where *R* denotes the firm's (net) revenue function, *N* its employment, *H* its number of hirings per period, and *V* its number of job vacancies. The wage level is denoted by *w* and recruitment costs are captured by the parameters α , as in Nickell (1986), and γ , as in Pissarides (2000). For simplicity we ignore firing costs.

Recruitment costs are in general composed of both *hiring costs* (α per hiring) and *vacancy costs* (γ per vacancy and period). We incorporate both hiring costs and vacancy costs in the model because, as we shall see in Section 4.4, not only the size but also the structure of recruitment costs matter.

Next we introduce a fundamental assumption in job matching theory, namely that the flow of a firm's hirings is proportional to its recruitment efforts as measured by its stock of vacancies,

(7) $$H = qV ,$$

where, in general, *q* depends on the state of the labour market. Thus, in contrast to Nickel (1986), where *all* hirings are instantaneous, we assume here that a firm has to generate vacancies (start recruitment processes) to obtain new employees, so that *no* hirings are instantaneous, as in Pissarides (2000).

The constant of proportionality *q* in (7) can be interpreted as the probability per week that one of the *V* vacancies leads to a hiring, so that the waiting time has an exponential distribution with expected value equal to $1/q$ weeks. Of course, in a stochastic environment such a direct link between the stock of vacancies and the flow of hirings as in (7) can only apply to a large firm exploiting the law of large numbers, as emphasized by Pissarides (2000 p. 68), or to a large group of (small) firms.

We shall analyze the effect of recruitment costs on employment on the assumption that firms take the wage level *w* as given when prices are set or, more precisely, as unaffected by vacancy costs. This is hardly restrictive, but note that if wages are reduced as a compensation for higher vacancy costs, then the effect of longer recruitment times on employment will be *less* than it is in the following analysis.

4.2. Profit maximization with recruitment costs

In a steady state we have $H = sN$, where *s* denotes the (constant) separation rate, and substituting this expression and $V = H/q = sN/q$ into (6) we obtain

(8) $$\pi = R(N) - wN - (\alpha + \gamma/q)sN \, .$$

Next we use the simple economic principles in Nickel (1986 p. 481) and argue as follows. According to (8) a unit increase in employment generates additional costs of $w + (\alpha + \gamma/q)s$ per period in equilibrium. But in order to obtain a new employee the firm also has to generate a vacancy for $1/q$ weeks (on the average). A unit increase in employment consequently also involves a once for all cost of $\alpha + \gamma/q$, or, equivalently, a flow cost of $r(\alpha + \gamma/q)$ per period, where *r* is the interest rate. It follows that

(9) $$R'(N) = w + (r + s)(\alpha + \gamma/q)$$

in equilibrium for a profit-maximizing firm.

4.3. Employment and prices

As emphasized by Layard, Nickel, and Jackman (1991 p. 341) for a non-competitive firm *without* recruitment costs, equation (9) is an equilibrium relationship: “It is not a labour demand function because prices are chosen jointly with employment”. This is also true for a representative competitive firm. For, with $R(N) = pG(N)$, where *G* denotes the production function and *p* the market price, condition (9) reduces to

(10) $$pG'(N) = w + (r + s)(\alpha + \gamma/q) \, ,$$

and assuming that the firm is one of *n* identical firms in a competitive industry, the market price and a firm’s employment are determined by equation (10) and the equation

(11) $$nG(N) = D(p) \, ,$$

where $D(\cdot)$ is the industry’s product-demand function.

Pissarides (2000) assumes that the marginal product of labour is constant. But then it is particularly clear that equation (10) should be interpreted *not* as a marginal productivity condition for employment, as in Pissarides (2000), but as a *price equation*. For, if $G'(N) = a$, equation (10) can be written as

(12) $$p = \left(w + (r + s)(\alpha + \gamma/q)\right)/a \, .$$

As emphasized by Pissarides elsewhere, in Pissarides (1984 p. 133), an equation like (10) with $G'(N) = a$ is basically a modification of the classical condition on wages under constant returns to scale. The marginal product of labour (*a*) exceeds the real wage (w/p) because firms need to cover their recruitment costs. And in equilibrium in a competitive economy prices adjust to marginal costs, including recruitment costs.

It follows from (10) and (11) that higher recruitment costs reduce employment by reducing sales through higher prices (in accordance with the principle of derived demand). Note, in particular, that the magnitude of the effect of recruitment costs on employment depends on how important recruitment costs are for price setting.

When calculating the price of a product, a firm calculates all the required inputs at input prices. And from (12) it follows that if it takes *h* hours to make a product, then this will add $w(1+m)h$ to its price, where the mark-up *m* is given by

$$m = \frac{(r+s)(\alpha + \gamma/q)}{w}. \tag{13}$$

If, for example, *w* = 360 000 SEK per year, *r* = 3 per cent per year, *s* = 36 per cent per year, $\alpha = 100000$ SEK per hire (including the cost of introduction and training of a new employee), $\gamma = 10000$ SEK per week, and $1/q = 4$ weeks, then $m = 0.152$. When calculating the cost of a product's labour input, the firm will consequently apply a mark-up equal to 15.2 per cent to the basic wage rate. And the contribution of the vacancy cost γ to this mark-up is 4.3 per cent.

4.4. The structure of recruitment costs

The effect of longer recruitment times on employment through the mark-up *m* depends on the distribution of recruitment costs between hiring costs α and vacancy costs γ, according to (13).

The structure of recruitment costs depends on the search strategy used by firms. This question has been addressed by, for instance, van Ours and Ridder (1992). Using vacancy data from the Netherlands they conclude that employer search is mostly non-sequential. Almost all applicants arrive during the first two weeks after the announcement of a vacancy, which suggests that resources spent on job advertising in most cases are concentrated to the beginning of the recruitment process. Hence these costs do not depend on the duration of the vacancy and are consequently part of the hiring cost α. The same is true if the firm is using a private employment agency and is paying the agency for its services per job match.

On the other hand, increasing recruitment times as measured by *V/H* may be due to a decreasing share of instantaneous hirings. This may imply a shift from instantaneous hirings with $\alpha = 0$ towards more costly search for job applicants with $\alpha > 0$. Thus, even if the recruitment mark-up *m* is independent of the vacancy duration for hirings preceded by vacancies, it may increase discontinuously for other hirings, implying a higher mark-up on the average.

During recruitment of replacements firms sometimes experience unfilled jobs. Does this also mean that the (opportunity) cost of unfilled jobs should be included as vacancy costs which affect the mark-up in (13)?

The answer is ‘no’, for the following reasons.

Recruitment activities comprise efforts to attract job applicants followed by selection. Equation (6) does not model the choice between different methods of recruitment, only the effect on employment of choosing costly recruitment. But an implicit assumption of this model is, of course, that firms only use methods of recruitment which are consistent with profit-maximizing behaviour. And such methods cannot include unfilled jobs, since having unfilled jobs (‘idle machines’) does not in itself attract job applicants.

In fact, equation (6) already excludes unfilled jobs, at least unfilled jobs defined as ‘dips in employment’ as elaborated in Section 5, because employment is assumed to be constant over time in (6). In other words, this section deals with the effect on employment of costly search on the simplifying assumption that firms completely control employment.

This may be a reasonable approach if unfilled jobs are rare and hard to predict, so that firms simply ignore them when prices are adjusted to recruitment costs. The approach may also be reasonable for employers who anticipate problems to keep employment constant, provided it also incorporates plans to use substitutes (including personnel from temporary work agencies) whenever substitutes are necessary during recruitment of replacements in order to avoid ‘dips in employment’. Of course, this also means that anticipated costs of the necessary substitutes must be added to vacancy costs. But note that only vacancy costs *above* the wage level *w* can be included, since the term wN in (6) already includes the basic costs of having posts occupied.

4.5. Hirings and vacancies

A firm stabilizes employment at its equilibrium level N by generating hirings at the rate of

(14) $$H = sN\ .$$

This is done by adjustment of the stock of vacancies according to (7) so that

(15) $$V = sN/q\ .$$

Adjustment to equilibrium can be instantaneous if V is assumed to be a choice variable (as in Pissarides 2000). Otherwise adjustment will be gradual. A firm that wishes to hire at the rate of sN announces new vacancies at the rate of sN. If $qV < sN$ the vacancy stock V will grow until $qV = sN$ and if $qV > sN$ the stock will decline until $qV = sN$.

Equations (14) and (15) for a firm, or a group of firms, show how hirings and vacancies adjust to variations in employment in equilibrium. It also shows that a change in q will change the number of vacancies. Hirings are affected only if employment is affected, and employment is affected only if the average duration of vacancies ($1/q$) affects employment by affecting recruitment costs.

Pissarides (2000 p. 9) rewrites equation (15) for all firms as

(16) $$u = \frac{s}{s + \theta q}\ ,$$

where $u = U/(N + U)$ and $\theta = V/U$ (where U is unemployment), and interprets the result as "an equation determining unemployment in terms of the two transition rates". This is an interpretation – in the Markovian framework we are accustomed to and seldom question – which suggests that a decline in q will raise unemployment, other things being equal. But other things will not be equal. Longer vacancy spells will increase the stock of vacancies and keep employment and hirings stable – unless higher recruitment costs reduce employment.

In this section we have studied indirect effects of longer recruitment times on employment through higher recruitment costs. We now turn to the study of direct effects of longer recruitment times on employment, effects which exist even when vacancy costs are negligible ($\gamma = 0$).

5. Direct effects of unfilled jobs on employment

So far we have assumed that a firm can keep its employment constant over time in a static environment. But a firm cannot in general avoid random fluctuations in its employment. These fluctuations in employment are mirrored by fluctuations in unfilled jobs. And these unfilled jobs exist because firms cannot always anticipate their need for new hires in time, and because it takes time to recruit personnel.

Fundamental to the following approach to unfilled jobs is the distinction between jobs and employment and the assumption that a firm does not choose its employment (or its unfilled jobs) but its jobs. For a chosen number of jobs (*D*), unfilled jobs (*V*) and employment ($N = D - V$) will fluctuate in a way which a firm cannot predict or control with certainty, even if it tries to avoid having unfilled jobs (and thus dips in employment) by appropriate recruitment activities.

More precisely, suppose first that hiring is instantaneous even if it is costly. Then a firm would choose a certain employment level *F* in equilibrium (according to the analysis in Section 4, with all recruitment costs captured by α). And employment would be constant over time in a static environment.

In general, however, a firm's employment level cannot always be kept constant, because separations cannot always be replaced instantaneously. In this case the firm cannot choose its employment path with certainty. Instead it has to choose an *employment strategy*. And assuming risk neutrality the firm's problem is to find an employment strategy which maximises its expected profits.

It is reasonable to assume that an optimal employment strategy for a firm in a static environment is to hire a certain number of workers (*D*), and then start recruiting every time a separation is anticipated, even if we cannot assume a priori that $D = F$. At any point in time, *D* will be equal to the sum of the number of filled jobs *N* ('satisfied labour demand') and the number of unfilled jobs *V* ('unsatisfied labour demand'), suggesting, first, that we define the firm's labour demand (or its number of jobs) as the employment strategy *D*, and, second, that this strategy can be observed as the sum of *N* and *V*. A formal model of a firm's decision problem in this case is presented in Appendix 2.

An equivalent definition of labour demand is that the number of jobs in a firm with N workers is equal to D if the firm is ready to employ $D-N$ properly qualified persons if they turn up. This is essentially the same as the definition of 'established posts' in Layard et al. (1991 p. 273), where the number of 'established posts' in a firm is equal to P if the firm will 'advertise' $P-N$ jobs when it has N workers. And the definitions are exactly the same if we assume, as Layard et al. (1991 p. 273) do, that "(i)f a firm advertises y vacancies, it must be ready to employ y (properly qualified) people if they turn up".

A firm which expects that it cannot keep employment constant at F but experiences random 'dips in employment' (that is, unfilled jobs) from time to time, may find it profitable to increase its number of jobs and choose $D > F$. This problem of 'genuine' vacancies was first addressed by Jackman, Layard, and Pissarides (1989 p. 378) and further discussed by Layard, Nickel, and Jackman (1991 p. 273), who argue that "firms might find it worth their while to declare more vacancies than they have productive jobs, if they expect that on average they will not be able to fill all their vacancies. Against this is the risk that they may then have to hire more people than they want, if people turn up."

But 'declare vacancies' must not be interpreted literally in this context. A large firm which hires new personnel regularly, for instance, may be able to select personnel from a spontaneous flow of job applicants without having to formally 'declare' or announce any vacancies at all. And if the firm finds a spontaneous flow inadequate, it may attempt to increase the flow by advertising its needs in general terms, without specifying the number of job applicants that will eventually be hired.

The point is that if a firm *has* $D-N$ unfilled jobs, it *is* ready to employ $D-N$ properly qualified people if they turn up, irrespective of the design of its recruitment activities. For, unless the firm hires such people, it will not maximise expected profits.

Friction reduces the average number of employees by the average number of unfilled jobs if $D = F$. But a gap between a firm's number of jobs and its average number of employees does not necessarily mean that friction reduces average employment *in equilibrium*. For, in equilibrium a firm should be able to anticipate such a gap, and then the firm can choose to increase its number of jobs to compensate for the gap, so that $D > F$. In this case friction would consequently increase not only the gap between the number of jobs and the average number of employees but also the number of jobs.

Layard et al. (1991 p. 274) derive a condition for when it is unprofitable for a firm to have an extra post ($D = F + 1$ instead of $D = F$) and find that this condition is satisfied for most firms. Moreover, this condition is almost the same as the condition for choosing $D = F$ in the model presented in Appendix 2.

Thus, we can assume that *firms do not react to friction in the hiring process by increasing the number of jobs*. It follows that the average number of unfilled jobs can be interpreted as a reduction of average employment caused by friction.

6. Conclusions

Large shifts of the matching function may indicate large changes in search efficiency, which have large effects on hirings and employment. But the effects of shifts of the matching function on hirings and employment cannot be estimated directly from regressions with hirings as the dependent variable. For what the relation between hirings, vacancies, and unemployment really shows is how *recruitment times* adjust to variations in labour supply and search efficiency. It follows that the proper specification of the relation between hirings (*H*), vacancies (*V*), and unemployment is the *duration function,* which shows how average recruitment times as measured by *V/H* depend on unemployment and other relevant variables.

Estimation of the duration function will show what factors which influence recruitment times and how large the effects are. This is an important first step. But how will variation in recruitment times affect employment?

We have seen that longer recruitment times may reduce employment if they increase recruitment costs so much that prices are raised and sales reduced so much that employment is also reduced. There is no information on this issue I am aware of. But if asked how variation in recruitment times will affect pricing (or employment), an employer might well answer that the effect is negligible.

On the other hand, longer recruitment times may also increase the risk of unfilled jobs. And unfilled jobs will reduce employment by creating a gap between the number of jobs and average employment. Moreover, the determinants of this gap may be identified by estimating a regression of the rate of unfilled jobs on unemployment and other relevant variables (including unemployment benefits). But this, of course, requires vacancy surveys which measure not only job vacancies but also unfilled jobs.

Appendix 1. Defining and measuring unfilled jobs

Measurement of unfilled jobs in enterprise surveys presupposes an operational definition in terms of specific questions to firms (or establishments). An example of a questionnaire designed to measure not only the number of job vacancies but also the number of unfilled jobs is presented in Figure 1. The example is taken from the Swedish vacancy survey, which started in July 2000, and which since January 2001 covers not only the private but also the public sector. It is statutory since July 2003.

The questionnaire in Figure 1 measures unfilled jobs defined as *unoccupied job openings which are available immediately*. It combines two basic principles. First, a direct question about the number of job openings is asked, as in the Dutch vacancy survey.[8] This approach rests on the assumption that employers interpret ’job openings’ as ‘job vacancies’ are defined in Section 2, particularly if a brief definition is given, as in Figure 1.[9] Second, unfilled jobs are measured indirectly, as unemployed workers are measured in labour force surveys, by a sequence of questions.

Thus, after the first question on the total number of job openings, questions are asked about how many of these job openings which are occupied on the reference day by retiring workers or substitutes or personnel from a temporary employment agency, and then those job openings which are *not* occupied are divided between ‘future’ and ‘current’ unoccupied job openings. Unfilled jobs are defined operationally as a subset of job openings obtained by eliminating first ’occupied job openings’ and then ’future unoccupied job openings’, defined more precisely as ‘unoccupied job openings which on the reference day are unoccupied because no work is wanted or planned until after the reference day’. The job openings which remain after these eliminations are ‘unoccupied job openings which are available immediately’.

‘Occupied job openings’ exist when, during recruitment of new workers, the corresponding jobs are occupied by retiring workers or substitutes until replacements or permanent personnel have been hired. Job openings which are occupied by retiring workers reflect employers’ ability to anticipate their need for new hires and certainly not ‘unsatisfied labour demand’. But job openings occupied by *substitutes* cannot

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[8] See Van Bastelaer and Laan (1994) for a detailed presentation of the Dutch vacancy survey, including its questionnaires.

represent completely satisfied labour demand, since the firms would not be recruiting new personnel if they did. On the other hand, workers with temporary employment who are looking for new jobs are not classified as unemployed, so a measure of unmet demand which corresponds to the usual measure of unmet supply should not include job openings occupied by substitutes.[10]

Figure 1. The questionnaire in the Swedish vacancy survey

1 JOB OPENINGS

Total number of job openings on the reference day: ……

A job is open if the employer has started to recruit a new worker from outside the firm on or before the reference day but has not yet hired one. During recruitment of new workers the corresponding jobs may be occupied or unoccupied according to the following questions.

2 JOB OPENINGS WHICH ARE OCCUPIED

a) Number of job openings which on the reference day are occupied by retiring workers who have not yet left the employer: ……

b) Number of job openings which on the reference day are occupied by substitutes or other temporary workers: ……

c) Number of job openings which on the reference day are occupied by consultants or personnel from a temporary employment agency: ……

3 JOB OPENINGS WHICH ARE UNOCCUPIED

a) Total number of unoccupied job openings: ……

b) Number of these unoccupied job openings which on the reference day are unoccupied because no work is wanted or planned until after the reference day: ……

Note: Unfilled jobs (unmet labour demand) are operationally defined as a subset of job openings obtained by eliminating first occupied job openings (1 – 2 = 3a) and then job openings which are not available until some time in the future (3a – 3b).

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[9] Since a 'vacancy' is an ambiguous term (which sometimes is interpreted as an unfilled job) it is wise to avoid it altogether in a business survey.

[10] See Farm (2003) for a more detailed discussion of the problem of defining and measuring unmet labour demand.

Appendix 2. Choosing the number of jobs in a stochastic environment

Consider a non-competitive or competitive firm in a model with one input and constant returns to labour (*a*). With instantaneous hiring the firm would simply adjust its production and employment to its sales (at the price *p* determined by the firm or the market) and replace separations instantaneously. Let *F* denote its optimal employment in this case. Then its profits per period are given by $(y-w)F$, where *y* denotes (constant) revenues per period and worker ($y = pa$) and *w* denotes the (constant) wage rate.

In general, however, a firm cannot instantaneously control its employment. Assuming that the firm cannot change its price during random fluctuations of its employment, its maximal sales are given by aF. If employment is less than *F* the firm will consequently loose sales and profits. If it is greater than *F* the firm will either produce more than it can sell or have idle (and costly) capacity. More precisely we assume that output cannot be stored, so that the firm's profits per period are given by

(1) $$\pi = (y-w)N \text{ if } N \le F,$$

(2) $$\pi = yF - wN \text{ if } N > F,$$

where *N* denotes the firm's employment,

(3) $$N = N_1 + N_2 + ... + N_D,$$

where $N_j = 1$ if job *j* is filled and $N_j = 0$ if it is unfilled.

Assuming that a filled job becomes unfilled with probability ρ per period, while an unfilled job gets filled with probability σ per period, and assuming in addition that $\{N_j\}$ are independent, *N* is a random variable with a binomial distribution with parameters *D* and $1-v$, where

(4) $$v = \frac{\rho}{\rho+\sigma}$$

can be interpreted as the firm's vacancy rate, since $E(V) = D - E(N) = Dv$.

Obviously it will never be optimal for the firm to choose $D < F$. Moreover, if $D = F$ then $N \le F$ and $\pi = (y-w)N$. It follows that the firm's expected profits are

(5) $$E(\pi) = (y-w)E(N) = (y-w)F(1-v) \text{ if } D = F.$$

Next we note that if $D = F + 1$ then

(6) $$E(\pi)=\sum_{k=0}^{F}(y-w)kP(N=k)+\left(yF-w(F+1)\right)P(N=F+1)=$$

$$=\sum_{k=0}^{F}(y-w)kP(N=k)+\left(y(F+1-1)-w(F+1)\right)P(N=F+1)=$$

$$=\sum_{k=0}^{F+1}(y-w)kP(N=k)-yP(N=F+1)=$$

$$=(y-w)E(N)-y(1-v)^{F+1}=(y-w)(F+1)(1-v)-y(1-v)^{F+1}.$$

Hence it will be unprofitable to have one extra post if

(7) $$(y-w)(F+1)(1-v)-y(1-v)^{F+1}<(y-w)F(1-v),$$

or, equivalently, if

(8) $$1-w/y<(1-v)^{F},$$

which is the same condition as in Layard et al. (1991 p. 274).

But the argument does not prove formally that more than one extra post is also unprofitable if (8) is true. However, if v is small the right hand side is approximately equal to $1-Fv$, so condition (8) is approximately equivalent to

(9) $$F<\frac{w/y}{v},$$

and it is at least possible to prove formally that

(10) $$D=F \text{ if } F<\frac{(w/y)(1-v)}{v}.$$

To prove (10) we begin by noting that since $\pi\le yF-wN$ for every N and $\pi<yF-wN$ if $N<F$ we have

$$E(\pi)<yF-wE(N)=yF-wD(1-v) \text{ if } D>F.$$

Hence $(E\pi)_{D>F}<(E\pi)_{D=F}$ if

$$yF-wD(1-v)<(y-w)F(1-v),$$

or if

$$vyF<w(D-F)(1-v).$$

And this inequality is obviously true for every $D>F$ if it is true for $D=F+1$, that is, if

$$vyF<w(1-v),$$

which is equivalent to (10).

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