Job Openings, Hirings, and Unmet Demand:
A New Approach to the Matching Function and the Beveridge Curve*

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Abstract: Firms *create* ‘vacancies’ in one sense (recruitment processes) in order to *avoid* ‘vacancies’ in another sense (unmet demand). The paper clarifies the different roles of these two concepts in labour market analysis, not only when interpreting Beveridge curves and matching functions, but also when analysing the effect of recruitment problems on employment at the firm level.

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1. Introduction

When using statistics on vacancies it is of vital importance to note that vacancies in practice can be defined not only as 'unfilled jobs' (as the dictionary tells us) but also as 'recruitment processes'. For instance, according to Holt and David (1966 p. 82):

> 'Firms recognize that a certain amount of time will be required to recruit, hire, and train new workers, or to recall old ones, i.e. there is a lead time in hiring workers. Consequently, new vacancies need to be created in anticipation of future needs, both to compensate for expected losses of the work force through quits, recruitments, and terminations during the lead time, and to build up the work force for higher production levels that may be desired in the future.'

Here it is obvious that 'creating vacancies' in anticipation of future needs does not mean creating *unfilled jobs* but starting recruitment activities or *recruitment processes*. And the purpose of these recruitment processes is to hire new workers in time, before the corresponding jobs become unfilled. In other words, firms *create* 'vacancies' in one sense (recruitment processes) in order to *avoid* 'vacancies' in another sense (unfilled jobs).

To emphasize this distinction, vacancies as recruitment processes will be called *job openings* in this paper, while vacancies as a measure of unmet labour demand will be called *unfilled jobs*. And the purpose of the paper is to clarify the different roles of these two concepts in labour market analysis, not only when interpreting Beveridge curves and matching functions, but also when analysing the effect of recruitment problems on employment at the firm level.

In the literature on vacancies an outward shift of the Beveridge curve or, equivalently, increased unemployment at a given vacancy rate, has been interpreted as an increase in 'maladjustment' in the labour market, beginning with Dow and Dicks-Mireaux (1958), or as a decline in the 'search effectiveness' of the unemployed, as in Jackman, Layard, and Pissarides (1989) and Layard, Nickell, and Jackman (1991). But more vacancies at a given unemployment rate may also suggest a rise in job turnover or job reallocation, as emphasized by, for instance, Abraham (1987 p. 230), Schager (1987 p. 33), Blanchard and Diamond (1989), Katz and Krueger (1999 p. 29), and Petrongolo and Pissarides (2000 p. 2).[1] In this paper we shall see how the interpretation of an outward shift of the Beveridge curve also depends on whether the Beveridge curve refers to job openings or unfilled jobs.

All vacancy surveys in the twentieth century have measured job openings and not unfilled jobs (unmet demand).[2] So the paper begins by discussing job openings.

Section 2 begins by emphasizing that there are job openings without hirings and hirings without job openings. It then formulates a basic relation between recruitment decisions and hirings, and shows how variations in job openings between quarters are determined by variations in recruitment decisions and job opening durations.

A key concept in the search and matching literature is the matching function.[3] However, Section 3 argues that the traditional matching function is incomplete. It governs the *speed* at which workers fill job openings – which of course is important enough - but does not explain how the *number* of hirings per period is affected by the matching process. A relation between hirings and recruitment decisions is offered as the missing link.

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[1] But the fact that an increase in the number of job vacancies can be indicative of increased turnover was pointed out already by Thomson (1966 p. 191).

[2] Partial exceptions include the Dutch annual vacancy survey 1980-1987, which measured occupied job openings and hence also *unoccupied* job openings, and the German vacancy survey by IAB which separates job openings *to be filled immediately* ('sofort zu besetzende Stellen') from job openings *to be filled later* ('später zu besetzende Stellen'). See van Bastelaer and Laan (1994 p. 18) and, for instance, IAB Kurzbericht Nr. 5/31.5.1999, which can be downloaded from http://www.iab.de. Note that unmet demand is measured by *unoccupied job openings to be filled immediately*, as emphasized, for instance, in the proposal by Eurostat (2000) to launch a vacancy survey in the member states. This excludes occupied job openings and job openings to be filled later, in the same way as unmet supply (unemployment) excludes job seekers with a job (on-the-job search) and job seekers without a job who cannot start working until later.

[3] See Petrongolo and Pissarides (2000) for an extensive survey.

In Section 4 we find that a stable relation between hirings, job openings and unemployment for quarterly data can be interpreted not only as a (traditional) matching function but also as a *vacancy function*, which explains the number of job openings in terms of a limited number of variables, provided that the number of hirings per quarter is interpreted as a measure of the number of recruitment decisions per quarter. We also find that this vacancy function facilitates the interpretation of movements of the *UV* point in a *UV* diagram.

Both shifts of the (traditional) matching function and shifts of the Beveridge curve for job openings (controlling for job turnover) reflect changes in the duration of job openings. Now, longer recruitment times undoubtedly suggest a decline in ‘search effectiveness’. But we want to be more precise. Let us here focus on the relation between recruitment times and unemployment and see how recruitment times contribute to unemployment by reducing employment.

Problems for firms to recruit workers may contribute to unemployment in three ways. Firstly, the matching of workers and jobs takes *time* which adds directly to unemployment, as suggested by almost every introductory text in macroeconomics or labour economics.[4] More precisely, as elaborated in Section 5, the time it takes to recruit workers adds to unemployment by making the number of employees less than the number of jobs. And the difference is the number of vacancies, provided only *unfilled jobs* are counted.

Secondly, longer recruitment times may reduce empbyment without generating unfilled jobs or raising wages but by raising *recruitment costs*, as in dynamic theories of labour demand, including Nickell (1986) and Pissarides (1990). In Section 6 we find that this effect on employment of longer recruitment times depends on: 1) the wage elasticity of labour demand; 2) how important total recruitment costs are relative to wages; and 3) how important 'variable' recruitment costs are relative to 'fixed' recruitment costs, where 'variable' recruitment costs are recruitment costs related to the time it takes to recruit personnel.

Thirdly, recruitment problems may reduce employment by raising *wages*. Section 7 focuses on this effect of recruitment problems on unemployment. It relates the classical concept of structural-frictional unemployment in Thirlwall (1969) to modern concepts of structural (equilibrium) unemployment in real-wage models such as Pissarides (1990). The basic idea is that recruitment problems by fuelling wage competition may reduce the number of jobs which gives 'tolerable' (or non-increasing) inflation. But note that firms have no reason to raise their wage offers if they manage to hire new workers in time to replace separations or begin expansions according to plan. Thus, when estimating wage equations with the vacancy rate as one of the independent variables, it is statistics on unfilled jobs (unmet demand) which is needed, not statistics on job openings.

Section 8 concludes the paper by emphasizing how important the development of statistics on unmet labour demand is for understanding the determinants of unemployment.

2. Job openings, recruitment decisions, and hirings

A firm's 'job openings' are related to its hirings and to its recruitment practices. Some hirings are made more or less directly, for example by recalling workers previously laid off. In other cases the firm has to attract job applicants by advertising its demand for personnel

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[4] See, for instance, Mankiw (1994 p. 122) or Ehrenberg and Smith (1994 p. 566).

in newspapers or other media, or by placing job orders with a public or private employment agency. And then vacancies understood as ‘recruitment processes’ arise, as discussed in, e.g., Burdett and Cunningham (1998). More precisely, a *job opening* begins when a firm starts to recruit a worker, and it ends when a worker offered the job accepts it (or when recruiting is discontinued for other reasons).

2.1. A basic relation

Can we assume that every hiring begins with a job opening? This assumption is implicitly made when attempts are made to estimate the total number of job openings (V) from the total number of hirings per month (H) and the average duration of job openings (T) for some part of the economy according to the formula

(1) $$V = HT,$$

as in, for instance, Abraham (1983) and Jackman, Layard, and Pissarides (1989). In this context the assumption is true by definition. But some hirings may occur without any preceding recruitment activities at all, for example when a job applicant contacts an employer who then decides to hire without having made efforts in the recent past to obtain job applicants (passive recruitment). And even if a recruitment process precedes a hiring, it can sometimes be so short that the distinction between the recruitment process and the hiring is negligible. Examples include recalls by phone calls of former employees.

Hirings with negligible or non-existent recruitment processes before hiring will be called *instantaneous hirings* in this paper.

Let R denote the number of *recruitment decisions* during a given period for a firm (or set of firms). Some of these decisions are associated with instantaneous hirings (aR), while the rest generates an inflow of job openings which to some degree (b) also end in hirings, so that the number of hirings (H) which (sooner or later) result from the recruitment decisions is given by the relation

(2) $$H = aR + b(1-a)R.$$

A firm's decisions to recruit people are determined by its desired net change of employment and its need to replace workers who, for various reasons, are leaving the firm. And the firm's desired net change of employment depends on current employment, adjustment costs, current revenues and costs (including recruitment costs), and expectations of future revenues and costs, as elaborated, for instance, in Nickell (1986). Equation (2) shows how the firm's actual hirings (H) are related to its recruitment decisions (R) and two parameters (a and b) characterizing the job matching process.

If all hirings are instantaneous, as in Nickell (1986), then a firm's hirings are completely determined by its recruitment decisions. Once a firm has decided how many workers it wants to recruit, it simply hires them (for instance by recalling former employees). This case is modeled by assuming that $a = 1$.

But the relation $H = R$ between hirings and recruitment decisions also follows from assuming that $b = 1$. This assumption, which always is made (implicitly) in the theoretical search literature,[5] means that all job openings sooner or later end in hiring. Some sample surveys by the Public Employment Service in Sweden in the beginning of the 1990's also suggest that the proportion of job openings which end in hiring is very high, and at least equal to 90 percent.[6]

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[5] See, for instance, Mortensen and Pissarides (1999).
[6] See Falk (1996). Results by van Ours and Ridder (1992 p. 145) suggest the same thing.

In general, however, the number of a firm's hirings is proportional to the number of its recruitment decisions,

(3) $$H = cR,$$

with a constant of proportionality, $c = a + (1-a)b$, which is less than 1 and which also may depend on the state of the labour market.

2.2. Stocks of job openings

The number of job openings at a particular point in time depends on how many job openings which have been generated in the past and how long each job opening lasts. Moreover, since the duration of a job opening is relatively short, the stock of job openings (V) for a firm (or group of firms) only depends on inflows and durations in the recent past. More precisely, as elaborated in Appendix 1, the stock of job openings is approximately determined by the inflow of job openings (F) *during the past quarter* and the average duration of job openings (T) *during the past quarter* according to the standard formula for flow equilibrium,[7]

(4) $$V = FT.$$

To see this intuitively, consider a large firm with an inflow of job openings equal to F per week during the past quarter and suppose (for simplicity) that all job openings have the same length, namely T weeks. At a given point in time the stock of job openings will then be equal to FT, which is equal to the number of job openings which started during the past T weeks, *provided that* $T \leq 13$. (If $T > 13$ the stock of job openings would also depend on

[7] Of course the standard formula for flow equilibrium applies to quarterly data for unemployment (or employment) only in equilibrium.

inflows of job openings in previous quarters.) And most vacancy durations are shorter than 13 weeks. In fact, typical values for the average duration of job openings reported in the literature are 2 - 8 weeks in Europe and 2 - 4 weeks in the USA.[8]

Next we note that the inflow of job openings is determined by the flow of recruitment decisions and the proportion of these decisions which are associated with job openings,

(5) $$F = (1 - a)R \ .$$

Now, combining equations (4) and (5) with our basic relation (3) between hirings and recruitment decisions we obtain the relation

(6) $$V = (1 - a)RT = \frac{1-a}{c}HT \ ,$$

which generalizes (1) to $a > 0$ and $c < 1$.

As noted above this relation has often been used to estimate the total number of job openings from data on total hirings and data on the average duration of job openings for some part of the economy. But equation (6) also shows how variations in job openings are determined by variations in recruitment decisions (R) and the average duration of job openings (T) - or by variations in *hirings* (H) and job opening durations, provided that hirings in this context are interpreted as a measure of recruitment decisions.

Note that the number of hirings during a given period is (approximately) equal to the number of recruitment decisions during the same period if the period is sufficiently long compared to the duration of job openings (and if c is close to 1). Now, while the duratio n of unemployment is measured in months (or years), the duration of job openings is usually measured in weeks, as noted above. This means that the number of hirings during a period

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[8] See, for instance, Muysken (1994 p. 7) for a survey of the duration of job openings in five European countries

should be a good measure of the number of recruitment decisions during the same period if the period is at least as long as a quarter. It also means that even if equation (6) does not apply to monthly data (except in equilibrium), it should apply to quarterly data on job openings, hirings, and job opening durations.

3. The matching function

Suppose now (for simplicity) that there are no instantaneous hirings ($a = 0$) and that all job openings end in hiring ($b = 1$). Then (6) reduces to $V = HT$, which can be written as $V = H/q$, where $q = 1/T$, or as

(7) $$H = qV.$$

Here q can be interpreted as the probability per week that one of the V job openings leads to a hiring (so that the waiting time has an exponential distribution with expected value equal to $1/q = T$). Of course, in a stochastic environment such a direct link between the stock of ongoing recruitment processes (V) and the flow of hirings (H) as in (7) can only apply to a large firm exploiting the law of large numbers, as emphasized by Pissarides (1990 p. 22), or to a large group of (small) firms.

Let us now apply equation (7) to the whole economy and assume that q depends on the state of the labour market, for instance $q = q(V/U)$, where U denotes unemployment, so that

(8) $$H = m(V, U),$$

1960-1990, and Blanchard and Diamond (1989 p. 3) for the USA.

where $m(V,U) \equiv Vq(V/U)$. In the search and matching literature equation (8) is interpreted as an *aggregate matching function*, showing how vacancies and unemployment as 'inputs' give rise to 'output' in the form of hirings.

Matching technologies of this form can be motivated by urn models in probability theory, where firms play the role of urns and workers the role of randomly chosen balls. But when summarizing the microfoundations behind the aggregate matching function, Petrongolo and Pissarides (2000 p. 6) conclude that: 'although there are several microeconomic models that can be used to justify the existence of an aggregate matching function, none commands universal support'.

On the other hand, equation (7) for a representative firm merely formalizes the idea that a firm in most cases (excluding instantaneous hirings) has to 'do something' (*V*) in order to recruit people. In other words, a hiring presupposes a job opening, that is, a recruitment process, and this recruitment process generates a hiring with some probability per week (*q*). It follows that at any given moment in time the number of hirings per week will be proportional to the number of ongoing recruitment processes, as (7) or (8) tells us. And of course the waiting times for new workers ($1/q$) in general depend on the state of the labour market.

So equation (8) is hardly problematic as a *conditional* matching function, which relates the flow of hirings to a *given* number of job openings. What is problematic is that as a matching function it is not *complete*. It captures some but not all of the search technology of the market, as we shall now see.

Purchases of goods or services in a market economy are generated by decisions to purchase, followed by search processes which, in general, are costly and time-consuming.

This also applies to the labour market. Thus, hirings are generated by firms' recruitment decisions. Some of these decisions are followed (almost) instantaneously by hirings. Other recruitment decisions are followed by recruitment processes which end by hirings after waiting times which in general are random and sometimes long. And some recruitment processes may end without hirings.

We want to model this relation between recruitment decisions and hirings. To do this, let $V(t)$ be the number of job openings at time t for a large group of firms. Moreover, let $F(t)$ and $O(t)$ denote the inflow intensity and outflow intensity of job openings at t (measured in thousands per week), and let $R(t)$ be the inflow intensity of recruitment decisions and $H(t)$ the outflow intensity of hirings. In general we then have

(9) $$H(t) = aR(t) + bO(t),$$

where (for simplicity) we assume that a and b are constant. Assume in addition that

(10) $$O(t) = qV(t),$$

where the outflow rate q in general depends on t, $q = q(t)$. It follows that

(11) $$H(t) = aR(t) + bq(t)V(t),$$

where $V(t)$ is determined by the linear differential equation

(12) $$V'(t) = F(t) - q(t)V(t).$$

It follows (see Appendix 1) that

(13) $$H(t) = aR(t) + bq(t)\left[e^{-Q(t)}V(0) + \int_0^t e^{-(Q(t)-Q(s))}F(s)ds\right],$$

where

(14) $$F(s) = (1-a)R(s),$$

and

(15) $$Q(s) = \int_0^s q(x)dx .$$

This is a complete description of the matching technology, where the traditional ('conditional') matching function enters as a building block through the last term in equation (11). Equation (13) shows how hirings depend on present and past recruitment decisions (R) and outflow rates of job openings (q). Of course the outflow rate in general also depends on the state of the labour market.

Moreover, for job openings the outflow rate is so large that only inflow intensities and outflow rates during the past quarter matter for the stock (see Appendix 1). As a first approximation we then have $V \approx F/q$ for quarterly data on job openings, so that for quarterly data on hirings (H) and recruitment decisions (R) we obtain from (11) and (14).

(16) $$H \approx aR + bq(1-a)R/q = cR .$$

This equation relates the number of recruitment decisions to the number of hirings.[9] The hiring rate c, which in general depends on the state of the labour market, tells us what proportion of the recruitment decisions which ends in hiring. This seems to be a parameter of fundamental interest, for different types of firms and different types of jobs, since it affects the arrival rate of offers which face job seekers.

We conclude that while the traditional matching function (8) governs the *speed* at which workers fill job openings (q) and hence the *duration* of the job openings ($1/q$), it does not show how the *number* of hirings per period is affected by the matching process. This aspect of the search technology is captured by the matching function in (16) for quarterly data and in general by equation (13).

4. The Beveridge curve for job openings

Movements of the *UV* point in a *UV* diagram for quarterly data on job openings and unemployment are easy to explain starting from equation (6), which says that job openings are proportional to hirings,

$$v = kh, \tag{17}$$

where v denotes the vacancy rate, $v = V/N$, h denotes the hiring rate, $h = H/N$, and $k = T(1-a)/c$ depends on the state of the labour market.

We first note that if, for instance, $k = k(u, v)$, where u is the unemployment rate, then (17) can be written as

$$v = f(h, u). \tag{18}$$

Thus, a stable relation between hirings, job openings and unemployment can be interpreted not only as a traditional ('conditional') aggregate matching function according to (8). It can also be interpreted as an *aggregate vacancy function*, which explains the number of job openings in terms of a limited number of variables, provided that the number of hirings per quarter is interpreted as a measure of the number of recruitment decisions per quarter.

As aggregate demand increases, hirings and job openings will increase as employment increases and unemployment falls, which implies a negative relationship between unemployment and job openings. More precisely, since

$$h = s + g, \tag{19}$$

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[9] Note that here H and R refer to the same period (quarter), while in equation (2) H is the number of hirings which *sooner or later* result from R.

where s denotes the separation rate and g the relative growth rate of employment during a quarter, $g = \dot{N}/N$, it follows from (17) that

(20) $$v = k(s + g),$$

where g is positive during an upswing, when unemployment decreases, and negative during a downswing. Thus, during a business cycle the *UV* point moves counter-clockwise around an equilibrium locus defined by the equation

(21) $$v = ks,$$

where, in general, not only $k = T(1-a)/c$ but also s depend on the state of the labour market.

It is useful to distinguish between the *observed UV* curve, which might be called the *UV loop*, defined by (20), and the *equilibrium UV* curve, often called the *Beveridge curve*, defined by (21). The negative relationship between job openings and unemployment during a business cycle which is actually observed in a *UV* diagram is always a part of the *UV* loop.

While shifts of the *UV* loop between business cycles may be due to shifts of the level of aggregate demand, shifts of the Beveridge curve reflect structural changes. More precisely, shifts of the Beveridge curve reflect changes in the average duration of job openings (T) *or* the separation rate (s) *or* the rate of instantaneous hirings (a) *or* the filling rate (b), or, if the parameters involved are not constant, shifts of the corresponding functions, for instance $T = T(u)$. Thus, controlling for job turnover (and assuming that the matching parameters a and b are constant), more job openings at a given unemployment rate (in equilibrium) means longer recruitment times.

5. Direct effects of recruitment times on employment

A firm does not always succeed in hiring a new worker in time to replace a separation or begin an expansion according to plan. And then the starting date of a new worker comes after the starting date desired by the employer, so that vacancies as unfilled jobs (unmet demand) arise. More efficient job matching may reduce such gaps between employment plans (jobs) and actual employment, and hence also increase employment for a given number of jobs.

Let V denote the number of unfilled jobs in a group of firms with employment N.[10] Then the vacancy rate V/N measures the direct effect of recruitment problems on employment for these firms. Of course some deviations from employment plans arise because some types of labour are simply not available when needed. Thus, the vacancy rate measures the effect on employment of all recruitment problems and not only pure matching problems.

Note that it is the rate of *unfilled jobs* (unmet demand) and not the rate of *job openings* (recruitment processes) which measures the direct effect of recruitment problems on employment. It is true that more *job openings* at a given unemployment rate may reflect longer durations of job openings (as we have seen in Section 4). But longer durations of job openings have direct effects on employment only if they give rise to unfilled jobs,[11] either by increasing the inflow of unfilled jobs or the duration of unfilled jobs.

The inflow rate of unfilled jobs p is related to, but in general smaller than, the separation rate s, $p \leq s$. It is sometimes assumed (for simplicity) that $p = s$, meaning that

[10] Of course not only employment and unemployment but also unmet demand can alternatively be measured in hours.

every separation gives rise to an unfilled job, or that firms have to create unfilled jobs in order to recruit workers.[12] But many separations are anticipated and replacements made before the corresponding jobs become unfilled, for instance by recalling workers previously laid off, and then $p < s$, particularly for large firms.[13] Some firms may even be able to avoid unfilled jobs altogether ($p = 0$), even if they generate job openings.

The duration of an unfilled job depends on the time it takes to recruit a worker, which includes not only the time it takes to attract job applicants but also the time it takes to select a suitable new employee, as emphasized by van Ours and Ridder (1992). It also depends on how long recruitment has been going on when the unfilled job begins, which means that it depends on a firm's ability to anticipate the need for new hires. Moreover, the duration of an unfilled job depends on the time it takes until a worker who has accepted a job offer also starts working.

In general both inflow and duration of unfilled jobs depend on the state of the labour market. The *Beveridge curve for unfilled jobs* shows how the number of unfilled jobs depends on unemployment (in a steady state), and hence also how unemployment depends on the number of jobs $D = N + V$. For instance, if

$$V = b_0 - b_1 U \qquad (b_0 > 0, b_1 > 0), \tag{22}$$

then

$$U = L - N = L - (D - V) = L - D + b_0 - b_1 U , \tag{23}$$

so that

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[11] The *indirect* effects of longer recruitment times are discussed in the next section.
[12] See, for instance, Pissarides (1990 p. 4), where it is assumed that 'only vacant jobs can engage in trade', and Blanchard and Diamond (1989 p. 9), where it is assumed that a vacancy is not advertised until the corresponding job becomes productive and unfilled.

$$U = \frac{1}{1+b_1}\left(L + b_0 - D\right). \tag{24}$$

Consider now an outward shift of the Beveridge curve or, equivalently, an *upward* shift (in a *UV* diagram with unemployment along the horizontal axis). Suppose more precisely that the number of unfilled jobs increases by ΔV for every unemployment rate, so that $\Delta b_0 = \Delta V$ (and $\Delta b_1 = 0$). Then $\Delta U = \Delta V / \left(1 + b_1\right)$ if $\Delta L = \Delta D = 0$. Thus, a decline in 'search effectiveness', as measured by more unfilled jobs at a given unemployment rate (ΔV), implies that, for a given number of jobs (*D*) and a given size of the labour force (*L*), unemployment increases by $\Delta V / \left(1 + b_1\right)$.

6. Effects of recruitment times on employment through recruitment costs

Longer recruitment times may reduce employment not only by reducing employment at a given number of jobs, but also by reducing the number of jobs through higher recruitment costs. To see this we follow Nickell (1986) and Pissarides (1990) and focus on the steady state.[14] Then there are no fires, and a firm's profit is given by

$$p = R(N) - wN - aH - gV, \tag{25}$$

where *R* denotes the firm's (net) revenue function, *N* its employment, *H* its number of hirings per period, and *V* its number of job openings. The wage level is denoted by w and recruitment costs are captured by the parameters a, as in Nickell (1986), and g, as in Pissarides (1990).

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[13] As emphasized, for instance, by van Ours and Ridder (1992 p. 140).

[14] More precisely we generalize the model for large firms in Pissarides (1990 p. 22) to include all recruitment costs (assuming that capital is fixed in the short run when firms recruit personnel), and use the short cut suggested by Nickell (1986 p. 481) to solve the optimization problem.

Recruitment costs are, in general, composed of both ‘fixed costs’ (a) and ‘variable’ costs (g). Fixed costs are independent of the length of the recruitment process. Suppose, for example, that a firm finds it necessary to announce a position in a newspaper. If this is done only once, or a predetermined number of times, it is a fixed cost or, in other words, a *hiring* cost. Then it is part of a. If, on the other hand, the firm advertises once a week until the vacancy is cancelled, then the cost is variable (dependent on the length of the recruitment process) and thus part of g. The same is true if the firm is using a private employment agency and is paying the agency for its services per week. But if the agency is paid per job match, then the cost is a hiring cost and thus part of a.

Next we introduce the (traditional) matching function,

(26) $$H = qV,$$

where q depends on the state of the labour market. Thus, the firm controls the inflow into employment (H) by varying its number of job openings (ongoing recruitment processes). The number of job openings equals the number of unfilled jobs if (and only if) firms have to create unfilled jobs (‘idle machines’) in order to recruit personnel.[15] But the point in this section is that recruitment problems may affect employment even when there are no unfilled jobs at all.

Assuming that a firm's separations are given by $S = sN$, where the separation rate s is constant, we have $H = sN$ in a steady state, and substituting this expression and $V = H/q = sN/q$ into (25) we obtain

(27) $$p = R(N) - wN - (a + g/q)sN.$$

Now we use the simple economic principles in Nickel (1986 p. 481) and argue as follows. A unit increase in employment generates not only additional costs of $w + \left(a + g/q\right)s$ per period in equilibrium but also a once for all cost of $a + g/q$, or, equivalently, a flow cost of $r\left(a + g/q\right)$ per period, where r is the interest rate. It follows that

$$R'(N) = w + (r+s)\left(a + g/q\right) \tag{28}$$

in equilibrium for a profit-maximizing firm.

As emphasized by Layard, Nickel, and Jackman (1991 p. 341) for a non-competitive firm (without recruitment costs), this equation is an equilibrium relationship: 'It is not a labour demand function because prices are chosen jointly with employment'. This is also true for a representative competitive firm. For with $R\left(N\right) = pF\left(N\right)$, where F denotes the production function, condition (28) reduces to

$$pF'\left(N\right) = w + \left(r+s\right)\left(a + g/q\right), \tag{29}$$

and assuming that the firm is one of n identical firms in a competitive industry, the market price p and a firm's employment are determined by equation (29) and the equation

$$nF\left(N\right) = D\left(p\right), \tag{30}$$

where $D\left(\cdot\right)$ is the industry's product-demand function.

Pissarides (1990) assumes that the marginal product of labour is constant, $F'\left(N\right) = a$, and then it is particularly clear that equation (29) should be interpreted as a *price equation.* As emphasized by Pissarides elsewhere,[16] an equation like (29) with $F'\left(N\right) = a$ is basically a modification of the classical condition on wages under constant

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[15] This assumption about the recruitment technology is perhaps most clearly spelled out in Pissarides (1985 p. 679), where 'a job may be thought of as a machine that could be operated by one worker', and where it is assumed that 'in order to engage in the search process that leads to job matches, (firms) must have an idle machine'.

returns to scale. The marginal product of labour (a) exceeds the real wage (w/p) because firms need to cover their recruitment costs. And in equilibrium in a competitive economy prices adjust to marginal costs, including recruitment costs.

Equation (30) then shows how employment in a competitive industry is determined by the market price, labour productivity, and the demand for the industry's products, while the number of job openings required to stabilise employment in a representative firm is determined by

(31) $$V = sN/q\,.$$

Moreover, since q in general depends on the state of the labour market, some adjustment may be needed until equations (29) and (30) are satisfied for every firm.

A change in wages will change market prices, sales, and employment. Assuming that the effect of a change in wages has a well-defined effect on employment, there is a well-defined wage elasticity of labour demand. It follows from (29) and (30) that the effects of recruitment costs on employment depends on this wage elasticity of labour demand and the relative importance of recruitment costs in total labour costs. Moreover, according to equation (29) the effect of a change in the average duration of job openings ($1/q$) on employment through recruitment costs depends on: 1) the wage elasticity of labour demand; 2) how important total recruitment costs are relative to wages; and 3) how important variable recruitment costs are relative to fixed recruitment costs.

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[16] See Pissarides (1984 p. 133).

7. Effects of recruitment times on employment through wage costs

The *UV* curve was introduced by Dow and Dicks-Mireaux (1958 pp. 4-5) to construct an index of excess demand in the labour market. Using this curve Thirlwall (1969) defined *demand-deficient unemployment* as that amount of unemployment which can be eliminated by increasing demand up to the point where the unemployment rate u is equal to the vacancy rate v. And that amount of unemployment which can not be eliminated by increasing demand up to the point where $u = v = m$, Thirlwall *called non demand-deficient unemployment.* Since then non demand-deficient unemployment (*m*) as defined by Thirlwall has often been interpreted as a measure of matching problems ('maladjustment') in the labour market during a given business cycle, a measure which often has been called *structural-frictional unemployment*.[17]

Now, Thirlwall (1969 p. 20) also argues that 'if the demand for labour was strong enough almost all unemployment could probably be eliminated (as in war time) but only at the cost of substantial upward pressure on wages and prices', suggesting that the concept of structural-frictional unemployment as non deficient-demand unemployment presupposes a constraint defining 'tolerable' inflation. On the other hand Thirwall (1969 p. 24) finds that opinions as to what constitutes 'tolerable ' inflation have differed widely. And having noted the ambiguity of a definition of demand-deficient unemployment based upon a subjective price norm, Thirlwall (1969 p. 27) proceeds by claiming that one 'obvious' criterion as an 'objective' dividing line would be the point of balance between supply and demand ($u = v$).

Of course 'equilibrium' defined as equality between supply and demand in a labour market is a classical point of departure in economics. But it is also closely associated with the notion that wages are increasing with excess demand ($v > u$), decreasing with excess supply ($u > v$), and stable when demand equals supply, or more precisely that

(32) $$\dot{w} = f(v-u), \quad f' > 0, \quad f(0) = 0,$$

where $\dot{w}$ denotes wage inflation, and where vacancies are defined as unfilled jobs (unmet demand).

On the other hand Hansen (1970 p. 23) argues that for wages to rise at the rate needed to keep prices constant, the rate of vacancies may in practice have to be higher or lower than the rate of unemployment, or more precisely that

(33) $$\dot{w} = f(v-u), \quad f' > 0, \text{ and } f(x) = \dot{w}_0 \text{ for some } x \le 0 \text{ or } x \ge 0,$$

where $\dot{w}_0$ is the wage-inflation rate needed to keep prices constant. This might happen, for instance, if (32) is true but in practice only an *ordinal* measure of unmet demand is available.[18]

In any case the basic idea in (33) is the same as in (32), namely that wage inflation is an increasing function of excess demand in the labour market, and that wage inflation is 'tolerable' only if excess demand is 'sufficiently low'. And combining this with the equation of the Beveridge curve for unfilled jobs, $v = g(u)$, we obtain structural-frictional unemployment determined by the system of equations

(34) $$f(v-u) = \dot{w}_0,$$

(35) $$v = g(u),$$

where $\dot{w}_0$ is the target wage-inflation rate.

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[17] See, for instance, Filer, Hamermesh and Rees (1996 p. 301).
[18] If, for instance, the rate of job openings is used as an index for the rate of unfilled jobs.

This concept of 'structural' unemployment is essentially the same as the concept of 'equilibrium' unemployment in modern unemployment theories, including, in particular, Pissarides (1990). To see this we first note that the real-wage model developed in Pissarides (1990) is based on a wage equation of the form[19]

(36) $$w/p = \overline{w} + b\, v/u \cdot$$

Moreover, the price equation (29), with $F'(N) = a$, can be written as

(37) $$w/p = a - (r+s)(a' + g'/q) \equiv (w/p)_0 ,$$

where $a' = a/p$, $g' = g/p$, and, in general, $q = q(v/u)$.

Combining these equations with the Beveridge curve we obtain an equilibrium unemployment rate determined by the system of equations

(38) $$\overline{w} + b\, v/u = (w/p)_0 ,$$

(39) $$v = g(u) ,$$

where $(w/p)_0$ is the target or ('feasible') real wage as defined by the price equation. In general this target depends on the state of the labour market, through $q = q(v/u)$, although this effect may not be very strong, as emphasized, for instance, by Layard et al. (1991 p. 13). Note that while (34) determines $v - u$, (38) determines v/u; and with $v - u$ or v/u determined, equation (39) determines unemployment.

In both of these approaches to structural unemployment, the basic idea is that recruitment problems may affect wage formation not only through unemployment but also through vacancies defined as unfilled jobs (unmet demand). Intuitively, while decreasing unemployment may increase wage pressure, unmet labour demand may add to wage drift

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[19] See, for instance, eq. (1.19) in Pissarides (1990).

(wage competition). Note that firms have no reason to raise their wage offers if they manage to hire new workers in time to replace separations or begin expansions according to plan. Thus, when estimating wage equations with the vacancy rate as one of the independent variables, it is statistics on unfilled jobs which is needed, not statistics on job openings.

8. Conclusions

We have seen, among other things, that the interpretation of an outward shift of the Beveridge curve crucially depends on whether the vacancy data refer to job openings (recruitment processes) or unfilled jobs (unmet demand). We have also seen that an outward shift of the Beveridge curve for *unfilled jobs* is a more direct measure of a decline in search effectiveness than an outward shift of the Beveridge curve for *job openings*. But surveys of unmet labour demand, as proposed for instance by Hoffmann (1999), are not yet available. They are badly needed.

Appendix 1. The relation between stocks and flows for vacancies

This appendix shows that the stock of vacancies interpreted as job openings (*V*) is approximately determined by the inflow of vacancies (*F*) during the past quarter and the average duration of vacancies (*T*) during the past quarter according to the standard formula for flow equilibrium,

(A.1) $$V = FT .$$

In general the duration of a vacancy is a stochastic variable and so is the stock of a firm's vacancies. Hence (A.1) cannot be literally true for every firm, particularly not for a small firm. Invoking the law of large numbers it can be approximately true for a large firm or a large group of small firms. So we focus on a large group of firms (for instance an industry or a country) and define T more precisely as

(A.2) $$T = 1/q ,$$

where q denotes the outflow rate for vacancies (outflow divided by stock) during the past quarter. For a small firm we may interpret (A.1) as an average over a large group of representative small firms.

Let $V(t)$ be the number of vacancies at time t, $t \geq 0$. Moreover, let $F(t)$ and $O(t)$ denote the inflow intensity and outflow intensity of vacancies at t (measured in thousands per month) and assume that

(A.3) $$O(t) = qV(t),$$

where the outflow rate q in general depends on t, $q = q(t)$. It follows that

(A.4) $$V'(t) = F(t) - q(t)V(t).$$

This is a linear differential equation for $V(t)$ which is solved by

(A.5) $$V(t) = e^{-Q(t)}V(0) + V^*(t), \quad t \geq 0,$$

where

(A.6) $$V^*(t) = \int_0^t e^{-(Q(t)-Q(s))} F(s)\,ds,$$

and

(A.7) $$Q(s) = \int_0^s q(x)\,dx .$$

Suppose now that $V^*(t)$ is of the same order of magnitude as $V(0)$. Then it follows from (A.5) that $V(t) \approx V^*(t)$ if $e^{-Q(t)}$ is negligible. In other words, if $e^{-Q(t)}$ is negligible, then the vacancy stock only depends on inflow intensities and outflow rates during the past t months, while the influence from the distant past through the vacancy stock t months ago is negligible.

Now, for vacancies the outflow rate is so large that only inflow intensities and outflow rates during the past 3 months matter for the stock. To see this we first note that the outflow rate for vacancies (outflow divided by stock) is seldom below 1.0 per month.[20] And when $q(s) \geq 1$ we obtain $Q(t) \geq t$ and $e^{-Q(t)} \leq e^{-t}$ if time is measured in months. Since $e^{-3} = 0.05$ it follows from (A.5) that

(A.8) $$0 \leq \frac{V(3) - V^*(3)}{V^*(3)} \leq 0.05k \text{ if } V(0) \leq kV^*(3),$$

so in this case the error in the approximation $V(3) \approx V^*(3)$ is less than 5 per cent if $k = 1$, and less than 10 per cent if $k = 2$, for example. (Of course the error is greater if $V(0)$ is much greater than $V^*(3)$.)

[20] See, for instance, Table 1 in Jackman et al. (1989) for stocks and flows of registered vacancies in Great Britain.

Suppose next (for simplicity) that the inflow intensity and the outflow rate are constant during a quarter. Measuring time in months from the beginning of a quarter we then obtain $Q(t) = qt$ and

(A.9) $$V(t) = e^{-qt}V(0) + (1 - e^{-qt})(F/q) \text{ if } 0 \le t \le 3,$$

which is a weighted average of the stock given by the equilibrium formula F/q and the initial stock $V(0)$. With $q \ge 1$ it follows, for example, that

(A.10) $$V(1.5) = w_1 V(0) + (1 - w_1) F/q \text{ where } w_1 \le 0.22,$$

Moreover,

(A.11) $$\int_0^3 e^{-qt}\, dt = (1/q)(1 - e^{-3q}) \approx 1/q,$$

so that the average value of the vacancy stock over three months is given by

(A.12) $$\overline{V} = \frac{\int_0^3 V(t)dt}{3} \approx \frac{(1/q)V(0) + (3 - 1/q) F/q}{3}.$$

We conclude that the vacancy stock during a quarter is determined mainly by the inflow intensity and the outflow rate during this quarter, while the influence from the past (through the vacancy stock in the beginning of the quarter) is relatively small.

References

Abraham, K. (1983), ‘Structural/Frictional vs. Deficient Demand Unemployment: Some New Evidence’, *American Economic Review*, 73, 708-724.

Abraham, K. (1987), ‘Help-Wanted Advertising, Job Vacancies, and Unemployment’, *Brookings Papers on Economic Activity*, 1:1987, 207-248.

Blanchard, O.J. and Diamond, P. (1989), ‘The Beveridge Curve’, *Brookings Papers on Economic Activity*, 1:1989, 1-76.

Burdett, K. and Cunningham, E.J. (1998), ‘Toward a Theory of Vacancies’, *Journal of Labor Economics*, 16, 445-478.

Dow, J.C.R. and Dicks-Mireaux, L.A (1958), ‘The Excess Demand for Labour’, *Oxford Economic Papers*, 10, 1-33.

Ehrenberg, R.G. and Smith, R.S. (1994), *Modern labor economics*, fifth edition, HarperCollins.

Eurostat (2000), ‘The Launching of a Vacancy Survey’, Doc. E1/EMPL/18/2000.

Falk, K. (1996), ’1995 års uppföljning av avrapporterade platser’, Pfuu 1996:2, AMS.

Filer, R., Hamermesh, D. and Rees, A. (1996), *The Economics of Work and Pay*, sixth edition, HarperCollins.

Hansen, B. (1970), ‘Excess Demand, Unemployment, Vacancies, and Wages’, *Quarterly Journal of Economics*, 84, 1-23.

Hoffmann, E. (1999), ‘Collecting Statistics on Imbalances in the Demand for Labour’, *Statistical Journal of the United Nations ECE*, 16, 105-121.

Holt, C.C. and David, M.H. (1966), ‘The Concept of Job Vacancies in a Dynamic Theory of the Labor Market’, in NBER (ed.), *The Measurement and Interpretation of Job Vacancies*, Colombia University Press.

Jackman, R., Layard, R. and Pissarides, C. (1989), ‘On Vacancies’, *Oxford Bulletin of Economics and Statistics*, 51, 377-394.

Katz, L. and Krueger, A. (1999), ‘The High-Pressure U.S. Labor Market of the 1990s’, *Brookings Papers on Economic Activity*, 1:1999, 1-97.

Layard, R., Nickell, S. and Jackman, R. (1991), *Unemployment. Macroeconomic Performance and the Labour Market*, Oxford University Press.

Mankiw, N.G. (1994), *Macroeconomics*, second edition, Worth Publishers.

Mortensen, D. and Pissarides, P. (1999), 'New Developments in Models of Search in the Labor Market', in Aschenfelter, O. and Card, D. (eds.), *Handbook of Labor Economics*, vol. 3B, North-Holland.

Muysken, J. (1994), 'Introduction', in Muysken, J. (ed.), *Measurement and Analysis of Job Vacancies,* Avebury.

Nickell, S.J. (1986), 'Dynamic Models of Labour Demand', in Ashenfelter, O. and Layard, R. (eds.), *Handbook of Labor Economics*, North-Holland.

Petrongolo, B. and Pissarides, C. (2000), 'Looking into the Black Box: A Survey of the Matching Function', CEPR Discussion Paper No. 2409.

Pissarides, C.A. (1984), 'Search Intensity, Job Advertising, and Efficiency', *Journal of Labor Economics*, 2, 128-143.

Pissarides, C.A. (1985), 'Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages', *American Economic Review*, 75, 676-690.

Pissarides, C.A. (1990), *Equilibrium Unemployment Theory*, Basil Blackwell.

Schager, N. (1987), *Unemployment, Vacancy Durations and Wage Increases: Applications of Markov Processes to Labour Market Dynamics*, IUI and Almqvist & Wiksell International.

Thirlwall, A.P. (1969), 'Types of Unemployment: With Special Reference to 'Non Demand-Deficient' Unemployment in Great Britain', *Scottish Journal of Political Economy*, 16, 20-49.

Thomson, W. (1966), 'Collection and Use of Job Vacancy Data in Canada', in NBER (ed.), *The Measurement and Interpretation of Job Vacancies*, Colombia University Press.

van Bastelaer, A. and Laan, J. (1994), 'The Job Vacancy Survey in the Netherlands', in Muysken, J. (ed.), *Measurement and Analysis of Job Vacancies*, Avebury.

van Ours, J.C. and Ridder, G. (1992), 'Vacancies and the Recruitment of New Employees', *Journal of Labor Economics*, 10, 138-155.